On the Dzhanibekov Effect: A Tumbling T-handle in Space

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1 Introduction

In the 1980s, when cosmonaut Vladimir Dzhanibekov uncrewed a T-shaped nut which has three axes of rotation, he noticed a peculiar behavior: the nut spun and then flipped. A team of mathematicians also independently noticed the effect in a tennis racket. They published their paper, named "The Twisting Tennis Racket".

Formally, this is known as the intermediate axis theorem, which goes as follows.

Theorem 1.1 (Intermediate Axis Theorem)**.** An object rotating about its intermediate axis with a very slight perturbation will undergo periodic flips in its orientation in the absence of external forces.

2 Proof

We first identify the three axes of rotation, as shown in fig. 1.

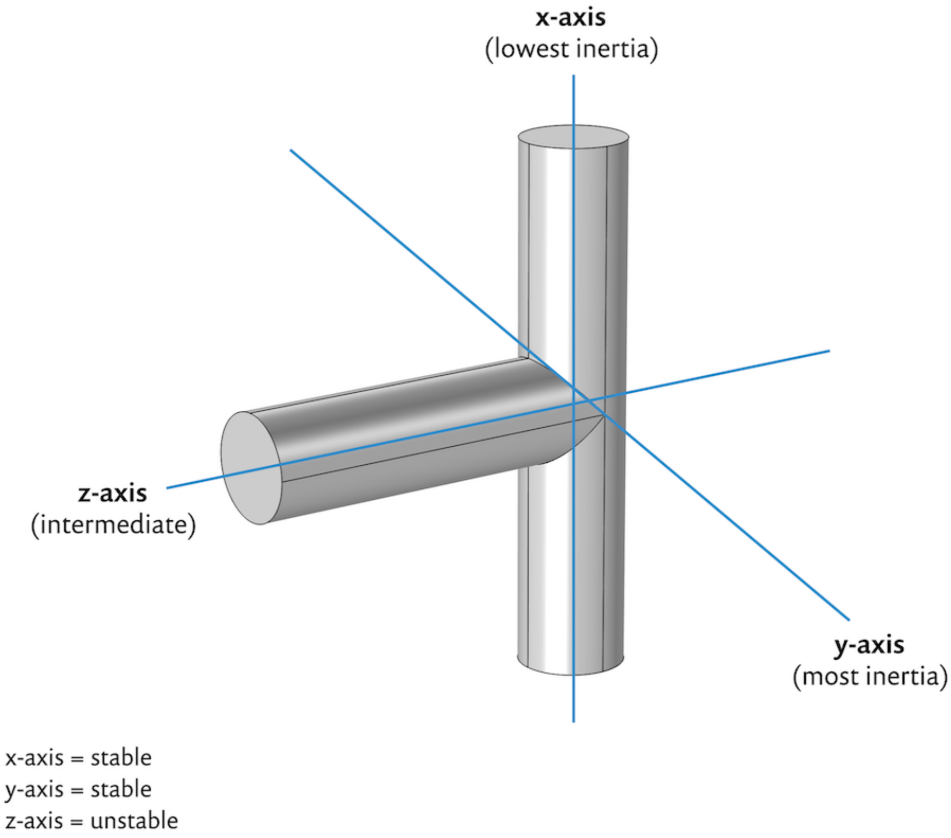

Figure 1: Axes of rotation [1]

The rotation of rigid bodies are described by **Euler's laws of motion**[1] [2], given by

$$\begin{aligned} I_1\alpha_1 - (I_2 - I_2)\omega_2\omega_3 &= \tau_1 \\ I_2\alpha_2 - (I_3 - I_1)\omega_3\omega_1 &= \tau_2 \\ I_3\alpha_3 - (I_1 - I_2)\omega_1\omega_2 &= \tau_3 \end{aligned} \tag{1}$$

where I_i denotes the moment of inertia along each of the three axes, α_i denotes the component of angular acceleration along the axes, ω_i denotes the component of angular velocity along the axes, τ_i denotes the torque. For our rigid body, we have $I_1 \neq I_2 \neq I_3$.

Remark. The proof of the Euler's laws of motion is rather complicated to be discussed in this text. It can be found here.

From $\tau = I\alpha$, the value for the moment of inertia I tells you how much torque you need to produce a given angular acceleration about that axis. The highest moment of inertia needs the most torque, while the lowest moment needs the least torque.

[1]These laws are an extension of Newton's laws, and extends them from Newton's point particles to rigid bodies.

If we consider the case where a rigid body is rotating freely around the 3-axis, we can examine under what conditions this rotation is stable or unstable. This is done by assuming small perturbations to the angular velocities around the 1- and 2-axes. After some manipulations of Euler's equations in eq. (1), we arrive at the following equation:

$$\dot{\alpha}_1 = -\left[\frac{(I_3 - I_2)(I_3 - I_1)}{I_1 I_2}{\omega_3}^2\right]\omega_1 \tag{2}$$

Since the part within the brackets is just a constant, let's call it k.

We have the following three cases:

1. I_3 is the largest moment of inertia.
2. I_3 is the smallest moment of inertia.
3. I_3 is neither, i.e. I_3 is the *intermediate axis*.

Cases 1 and 2:

It is easy to see that k is positive. This gives us

$$\ddot{\omega}_1 = -k\omega_1 \tag{3}$$

using the fact that α is the first time derivative of ω.

Looks familiar? This is the equation for simple harmonic motion. This is a stable motion, meaning that a small perturbation will not bring the body out of its equilibrium.

Case 3:

k is negative, giving us a positive overall constant. This equation is unstable.

References

[1] Julia Abrams. https://www.comsol.com/blogs/why-do-tennis-rackets-tumble-the-dzhanibekov-effect-explained/, 2020.

[2] Physics LibreTexts. https://phys.libretexts.org/Bookshelves/Classical_Mechanics/Classical_Mechanics_(Tatum)/04%3A_Rigid_Body_Rotation/4.05%3A_Euler's_Equations_of_Motion.