H2 Physics

Ryan Joo

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Abstract

This set of notes follows the Singapore GCE A-Level H2 Physics syllabus.

Definitions

Term	Definition
Base quantity	Physical quantity that cannot be defined in terms of other quantities.
Base unit	Unit which is defined without referring to other units.
Derived quantity	Physical quantity derived from base quantities, can be expressed in terms of product and/or quotient of base quantities.
Derived unit	Unit derived from base units, can be expressed in terms of products and/or quotients of base units.
Scalar quantity	Only has magnitude but no direction.
Vector quantity	Has both magnitude and direction.
Displacement	Distance moved in a specific direction.
Velocity	Rate of change of displacement with respect to time.
Acceleration	Rate of change of velocity with respect to time.
Projectile motion	Motion due to uniform velocity in one direction, uniform acceleration in a perpendicular direction.
Newton's 1st law of motion	A body at rest will remain at rest, a body in motion will remain in motion at constant velocity, in absence of external resultant force.
Newton's 2nd law of motion	Rate of change of momentum is directly proportional to external resultant force acting on it, and occurs in the direction of the external resultant force.
Newton's 3rd law of motion	When body A exerts a force on body B, body B exerts force of the same type, equal in magnitude, opposite in direction on body A.
Inertia	Reluctance of a body to change its state of rest or uniform motion in a straight line, due to mass.
Linear momentum	Product of mass and velocity.
Impulse	Product of constant force and time interval for which the constant force acts.
Principle of conservation of momentum	Total momentum of a system of bodies is constant, provided no external resultant force acts on the system.
Hooke's law	Force is directly proportional to extension of spring, provided that elastic limit has not been exceeded.
Centre of gravity	A single point where the entire weight of the object may be taken as acting at.

Term	Definition
Moment of a force	Product of magnitude of the force and perpendicular distance of line of action of force from pivot point.
Couple	A pair of forces of equal magnitude but acting in opposite directions whose lines of action are parallel but separate.
Torque of a couple	Product of one of the forces and the perpendicular distance between the forces.
Principle of moments	When a system is in equilibrium, sum of clockwise moments about any axis must be equal to sum of anticlockwise moments about the same axis.
Upthrust	Vertical upward force exerted by the surrounding fluid when a body is submerged (fully or partially) in a fluid.
Archimedes' Principle	Upthrust is equal in magnitude, opposite in direction to the weight of fluid displaced by the body.
Work done	Product of force and displacement in the direction of the force.
Principle of conservation of energy	Energy can neither be created nor destroyed, but can be transformed from one form to another, and transferred from one body to another. Total energy in a closed system is always constant.
Power	Rate at which work is done; rate at which energy is transferred.
Radian	Unit of angular measure, defined as the angle subtended at the centre of a circle by an arc of a length equal to the radius of the circle.
Newton's law of gravitation	Gravitational force of attraction between two point masses is directly proportional to the product of masses and inversely proportional to square of separation between their centres.
Gravitational field	Region of space where a mass experiences gravitational force.
Gravitational field strength (at a point)	Gravitational force per unit mass exerted on a small test mass placed at that point.
Gravitational potential energy (at a point)	Work done by an external force in bringing a small test mass from infinity to that point.
Gravitational potential	Work done per unit mass by external force in bringing small test mass from infinity to that point.
Geostationary satellite	Satellite that appears stationary when observed from a fixed location on Earth.
Thermal equilibrium	When two objects in thermal contact are in thermal equilibrium, there is no net heat transfer between them. They are at the same temperature.
Zeroth law of thermodynamics	If objects A and B are separately in thermal equilibrium with a third object C, then A and B are also in thermal equilibrium with each other.
Ideal gas	Hypothetical gas that obeys equation of state $pV = nRT$ perfectly for all pressure p, volume V, amount of substance n, temperature T.
Specific heat capacity	Thermal energy per unit mass per unit temperature change.
Specific latent heat	Thermal energy per unit mass required to change its state at constant temperature.
Internal energy	Sum of kinetic energy due to random motion of molecules, and potential energy due to intermolecular forces of attraction.
First law of thermodynamics	Internal energy of system depends only on its state. Internal energy of system is sum of heat supplied to system and work done on system.
Amplitude	Magnitude of maximum displacement from equilibrium position.
Period	Time taken for one complete oscillation.

Term	Definition
Frequency	Number of complete cycles per unit time.
Angular frequency	Measure of the rate of change of phase angle of the body's motion with time with respect to the centre of its oscillation.
Phase	Angle which gives a measure of the fraction of a cycle that has been completed by the oscillating particle or wave.
Phase difference	Angle which gives a measure of how much one oscillation is out of step with another.
Simple harmonic motion	Acceleration is directly proportional to displacement from a fixed point, and acceleration is in opposite direction to displacement.
Damping	Process where energy is lost from system as a result of dissipative forces.
Damped oscillation	Oscillation in which there is a continuous dissipation of energy to surroundings, such that total energy in system decreases with time, hence amplitude decreases with time.
Forced oscillation	Continual input of energy by external applied force to an oscillating system, to compensate energy loss due to damping, in order to maintain amplitude of oscillation.
Natural frequency	Frequency at which a body oscillates after initial disturbance.
Resonance	Resulting amplitude becomes maximum when driving frequency of external driving force equals to natural frequency of the system. Maximum transfer of energy from driving system to driven system.
Progressive wave	Wave in which energy is carried from one point to another by means of vibrations or oscillations within the waves, without transporting matter.
Displacement	Distance in a specific direction of a point on wave from equilibrium position.
Amplitude	Maximum displacement of any point on wave from equilibrium position.
Period	Time taken for one complete oscillation of a point in the wave.
Frequency	Number of oscillations per unit time of a point on the wave.
Wavelength	Minimum distance between any two points of the wave with the same phase at the same instant.
Wave speed	Speed with which energy is transmitted by wave.
Wavefront	Imaginary line or surface joining points that are in phase.
Transverse wave	Direction of vibration of particles perpendicular to direction of transfer of energy of wave.
Longitudinal wave	Direction of vibration of particles parallel to direction of transfer of energy of wave.
Intensity	Rate of energy transmitted per unit area perpendicular to direction of wave velocity.
Polarisation	Oscillations of wave are confined to only one direction, in the plane normal to the direction of transfer of energy of wave.
Principle of superposition	When two or more waves of same type meet at a point at the same time, displacement of resultant wave is vector sum of the displacements of individual waves at that point at that time.
Node	Region of destructive superposition where two waves meet antiphase. Displacement is permanently zero (or minimum amplitude).
Antinode	Region of constructive superposition where two waves meet in phase. Displacement is maximum amplitude.

Term	Definition
Stationary wave	Two progressive waves of the same type of equal amplitude, equal frequency, equal speed travel in opposite directions, meet, and undergo superposition with each other.
Diffraction	Spreading of waves at edge of obstacle, or through slit, so that waves do not travel in straight lines.
Coherent waves	Constant phase difference (with respect to time).
Interference	Superposition of coherent waves which results in change in overall intensity.
Constructive interference	When two waves meet in phase at a point, resultant displacement is the sum of magnitudes of individual displacements of the two waves.
Destructive interference	When two waves meet antiphase at a point, resultant displacement is the difference of magnitude of individual displacements of the two waves.
Rayleigh criterion	For two patterns to be just resolved, the central maximum of one must lie on the first minimum of the other.
Coulomb's law	Electric force between two point charges is proportional to product of charges and inversely proportional to square of separation.
Electric field	Region of space where a charge experiences an electric force.
Electric field strength (at a point)	Electric force per unit positive charge on a small test charge placed at that point.
Electric potential	Work done per unit positive charge by an external force in bringing a small test charge from infinity to that point.
Electric current	Rate of flow of charge.
Potential difference	Work done per unit charge when electrical energy is converted into non-electrical energy when the charge passes from one point to the other.
Electromotive force	Work done per unit charge when non-electrical energy is converted into electrical energy when the charge is moved around a complete circuit.
Resistance	Ratio of potential difference across component to current flowing through it.
Ohm's law	Current flowing through conductor is directly proportional to potential difference applied across it, provided that physical conditions remain constant.
Magnetic field	Region of space where magnetic pole / current-carrying conductor / moving charge particle will experience a force.
Magnetic flux density	Force acting per unit length of conductor which carries unit current and is of right angles to magnetic field. [Units: Tesla (T)]
Magnetic flux	Product of area and component of magnetic flux density perpendicular to the area. [Units: Weber (Wb)]
Magnetic flux linkage	Product of magnetic flux passing through the coil and number of turns on the coil.
Faraday's law	Magnitude of induced e.m.f. is directly proportional to rate of change of magnetic flux (linkage).
Lenz's law	Induced e.m.f. (or current) is in a direction so as to produce effects which oppose the change in magnetic flux (linkage).
Period	Time taken for the current to undergo one complete cycle.
Frequency	Number of complete cycles undergone by the current per unit time.
Angular frequency	Frequency in terms of radians per unit time rather than cycles per unit time.
Peak value	Maximum absolute value of a.c. (voltage) in either direction of zero value in a periodic cycle.

Term	Definition
Root mean square value	Value of steady direct current (voltage) which would dissipate heat at the same average rate in a given resistor.
Peak-to-peak value	Difference between maximum and minimum values of current within one cycle.
Ideal transformer	Device that changes a high alternating voltage (at low current) to low alternating voltage (at high current), and vice versa, with • no power loss (input power = output power) • no flux leakage (same magnetic flux passes through each turn of both primary and secondary coil)
Photon	Discrete packet (or quantum) of energy of EM radiation.
Photoelectric effect	Emission of electrons from metal surface when EM radiation of sufficiently high frequency is incident on it.
Heisenberg uncertainty principle	If measurement of position of particle is made with precision Δx and simultaneous measurement of momentum in x-direction is made with precision Δp, the product of these two uncertainties can never be smaller than h.
Isotope	Atom of the same element with same number of protons but different number of neutrons in nucleus.
Binding energy of nucleus	Minimum energy required to completely separate the nucleons (in nucleus) to infinity. Equivalently, energy released when nucleons come together (to form nucleus) from infinity.
Fission	Heavy nucleus with lower BE/nucleon is broken up into multiple lighter and more stable nuclei, each with higher BE/nucleon, with the release of energy.
Fusion	Multiple light nuclei with lower BE/nucleon combine into one heavier and more stable nucleus, with higher BE/nucleon, with the release of energy.
Radioactive decay	Spontaneous and random process where unstable nucleus changes into a different nuclide, emitting alpha, beta and/or gamma ray photons, with the release of energy, and formation of a more stable nucleus. • **Spontaneous**: occur without external stimuli e.g. changes in temperature, pressure • **Random**: not possible to determine which / when a particular radioactive nucleus will decay (experimentally, actual decay deviates from a smooth exponential graph with fluctuations in between)
Activity	Number of decays per unit time. [Units: s^{-1} or Bq]
Decay constant	Probability of decay of nucleus per unit time.
Half life	Time taken for quantity to reduce to half its initial value, on average.

Part I

Measurement

§1 Measurement

§1.1 Physical quantities and SI units

Base units:

Quantity	Unit
mass	kilogram (kg)
length	metre (m)
time	second (s)
current	ampere (A)
temperature	kelvin (K)
amount of substance	mole (mol)

Remark. Base units are not to be confused with **SI units**, which refer to the set of standard units that are commonly used. For instance, the SI unit for frequency is Hz, but the base unit is s^{-1}.

List of **prefixes**:

Prefix	Symbol	Factor
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}

Estimates of physical quantities:

- Diameter of nucleus $\approx 10^{-15}$ m
- Diameter of atom $\approx 10^{-10}$ m
- Diameter of earth $\approx 10^{7}$ m

§1.2 Dimensional analysis

Homogeneous equation: all quantities have the *same units* (use SI base units to check the homogeneity of physical equations) [1]

§1.3 Scalars and vectors

- **Vector addition**
  1. Triangle method
  2. Parallelogram method
- **Vector subtraction**

  Used to determine *change* in vector quantity.
- **Resolving vector**

  Represent a vector as two perpendicular components

Remark. Use of trigonometry, sine rule, cosine rule are relevant.

[1] A physically correct equation must be homogeneous; a homogeneous equation may not be physically correct. Some reasons include:
- Value of dimensionless factor may be incorrect.
- Missing or extra terms that may have the same unit.

§1.4 Errors

Systematic error	Random error
Error where repeating the measurement under the same conditions yields all measurements bigger or smaller than true value.	Error where repeating the measurement under the same conditions yields all measurements scattered about mean value.
Same magnitude and sign	Different magnitudes and signs
Can be eliminated by careful design of experiment, good experimental techniques.	Cannot be eliminated, but can be reduced by repeating measurements and averaging readings by plotting a best fit line for data points.
E.g. poorly calibrated instrument, instrumental zero error, human reaction time, parallax error	E.g non-uniformity of wires, instrument sensitivity, fluctuations in the testing environment (temperature, wind), irreproducible readings (repeat timing for 20 oscillations)

Accuracy	Precision
Degree of agreement between measurements and true value.	Degree of agreement among a series of measurements.
High accuracy is associated with small systematic error; mean value is close to true value.	High precision is associated with small random error; small scattering of readings about mean value.
Graphically, line of best fit does not pass through the origin.	Graphically, data points do not lie on a straight line, but scattered around the line of best fit.

§1.5 Uncertainties

Given a measurement R.

- **Actual uncertainty** is denoted as ΔR.
- **Fractional uncertainty** is given by $\frac{\Delta R}{R}$.
- **Percentage uncertainty** is given by $\frac{\Delta R}{R} \times 100\%$.

Remark. When there are more quantities, uncertainty increases.

Given measurements R, A, B, and coefficients m, n.

- For addition and subtraction where $R = mA + nB$, add or subtract **actual** uncertainties:

$$\Delta R = |m|\Delta A + |n|\Delta B \tag{1}$$

- For multiplication and division where $R = A^m B^n$, add or subtract **fractional** uncertainties:

$$\frac{\Delta R}{R} = |m|\frac{\Delta A}{A} + |n|\frac{\Delta B}{B} \tag{2}$$

- Use **First Principle** to deal with complex expressions

$$\begin{aligned}\Delta R &= \frac{R_{\text{max}} - R_{\text{min}}}{2} \\ &= R_{\text{max}} - R \\ &= R - R_{\text{min}}\end{aligned}$$

Part II
Newtonian Mechanics

§2 Kinematics

Velocity:

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} \tag{3}$$

Acceleration:

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \tag{4}$$

Graphically,

- displacement-time graph: gradient is velocity;
- velocity-time graph: gradient is acceleration, area under graph is displacement;
- acceleration-time graph: area under graph is velocity.

§2.1 Rectilinear motion

The following **equations of motion** only hold for *uniformly accelerated motion in a straight line.*

$$v = u + at \tag{5}$$

$$s = \frac{1}{2}(u+v)t \tag{6}$$

$$s = ut + \frac{1}{2}at^2 \tag{7}$$

$$v^2 = u^2 + 2as \tag{8}$$

See derivation

§2.2 Projectile motion

Analyse horizontal motion in the x-direction, and vertical motion in the y-direction separately. Resolving velocity into components,

$$v_x = v\cos\theta, \quad v_y = v\sin\theta.$$

This gives

$$v = \sqrt{{v_x}^2 + {v_y}^2}, \quad \theta = \tan^{-1}\frac{v_y}{v_x}.$$

Horizontal motion: no acceleration, so horizontal velocity remains constant

$$\begin{aligned} v_x &= u_x \\ s_x &= v_x t \end{aligned}$$

Vertical motion: acceleration due to gravity, so vertical velocity changes. Taking upwards as positive,

$$\begin{aligned} v_y &= u_y - gt \\ s_y &= \frac{1}{2}(u_y + v_y)t \\ s_y &= u_y t - \frac{1}{2}gt^2 \\ {v_y}^2 &= {u_y}^2 - 2gs_y \end{aligned}$$

You can derive time of flight,

$$\boxed{t_{\text{flight}} = \frac{2u\sin\theta}{g}}$$

maximum height,

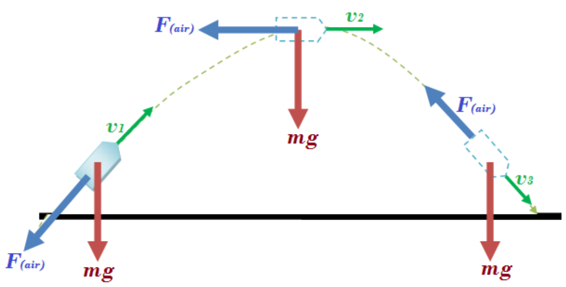
$$\boxed{h = \frac{u^2\sin^2\theta}{2g}}$$

range,

$$\boxed{R = \frac{u^2\sin 2\theta}{g}}$$

and range is maximum when $\theta = 45°$, $R_{\max} = \dfrac{u^2}{g}$.

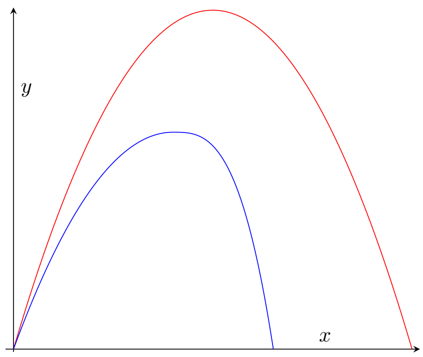
Effect of air resistance (air resistance opposes motion, increases as speed increases):



	Air resistance is negligible	Air resistance is not negligible
Horizontal velocity	Remains unchanged no horizontal deceleration	Decreases at a decreasing rate air resistance decreases with time due to decreasing horizontal velocity
Vertical velocity	Decreases at constant rate resultant force W is constant, so net downward acceleration is constant	Decreases at a decreasing rate resultant downward force decreases, so acceleration decreases
Time of flight	$t_u = t_d$	$t_d > t_u$ net downward acceleration due to $W - F_{\text{air}}$ is smaller than net upward deceleration due to $W + F_{\text{air}}$

Characteristics of path of object with non-negligible air resistance:

1. Lower maximum height, displaced to the left
2. Asymmetrical shape
3. Shorter range



Derivations

Equations of motion

$$\boxed{v = u + at}$$

Derivation. This is from the definition of acceleration. □

$$\boxed{s = \frac{1}{2}(u + v)t}$$

Derivation. Computing the area under a velocity-time graph (which has the shape of a trapezoid) gives the displacement. □

$$\boxed{s = ut + \frac{1}{2}at^2}$$

Derivation. Substitute eq. (5) into eq. (6) to eliminate v. □

$$\boxed{v^2 = u^2 + 2as}$$

Derivation. Rewrite eq. (5) to give $t = \dfrac{v - u}{a}$ which we can substitute into eq. (6) to eliminate t. □

§3 Dynamics

When drawing a **free body diagram**:

- Draw all external forces acting only on the chosen system.
- Do not draw in the resultant force.

§3.1 Newton's laws of motion

Be familiar with Newton's three laws of motion.

Newton's 2nd law of motion:

$$\sum F = \frac{\mathrm{d}p}{\mathrm{d}t} \tag{9}$$

Remark. Using product rule, we have

$$\sum F = \frac{\mathrm{d}(mv)}{\mathrm{d}t} = m\frac{\mathrm{d}v}{\mathrm{d}t} + v\frac{\mathrm{d}m}{\mathrm{d}t}.$$

Note that $\sum F = ma$ holds only when mass is constant.

Remark. Resultant force and acceleration act in the same direction, as a result of $\sum F = ma$.

Newton's 3rd law of motion:

$$F_{AB} = -F_{BA} \tag{10}$$

§3.2 Linear momentum and its conservation

Linear momentum:

$$p = mv \tag{11}$$

Impulse:

$$J = \int F \, \mathrm{d}t = F_{\text{avg}}\Delta t \tag{12}$$

Graphically, impulse is the area under a force-time graph.

For collisions, the area under both graphs (representing impulse of force on each object by the other) must be the same, as linear momentum is conserved.

Impulse–Momentum Theorem: impulse applied is equal to change in momentum.

$$J = \Delta p = p_f - p_i \tag{13}$$

Principle of conservation of momentum:

$$\sum F = 0 \implies J = 0 \implies p_i = p_f \tag{14}$$

§3.3 Collision (1-dimensional)

Types of collisions:

1. **Elastic**
2. **Inelastic**
3. **Perfectly inelastic**

Linear momentum is always conserved for every type of collision:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \tag{15}$$

Kinetic energy is conserved only for elastic collisions, from which we can derive relative speed of approach = relative speed of separation (r.s.a. = r.s.s.):

$$u_1 - u_2 = v_2 - v_1 \tag{16}$$

Kinetic energy is not conserved for inelastic collisions as energy is lost from systemm, so r.s.a > r.s.s.

$$u_1 - u_2 > v_2 - v_1$$

§4 Forces

§4.1 Types of force

Hooke's Law:

$$F = kx \tag{17}$$

where k is spring constant.

For springs in **parallel**, extension of all springs is the same, total force is sum of forces acting on all springs:

$$k_{\text{eff}} = \sum_i k_i. \tag{18}$$

For springs in **series**, force acting on all springs is the same, total extension is the sum of extensions of all springs:

$$\frac{1}{k_{\text{eff}}} = \sum_i \frac{1}{k_i} \tag{19}$$

Elastic potential energy stored in object when it undergoes deformation (extended or compressed).

$$U = \frac{1}{2}Fx = \frac{1}{2}kx^2 \tag{20}$$

Graphically, EPE is area under force-extension graph:

$$W = \int F \, \mathrm{d}x \, .$$

§4.2 Turning effects of forces

Moment of a force:

$$M = F \times \perp d \tag{21}$$

Torque of a couple:

$$\tau = F \times \perp d \tag{22}$$

Remark. A couple is a pair of forces which produces *rotation* only.

Principle of moments:

$$\sum \text{clockwise moments} = \sum \text{anticlockwise moments} \tag{23}$$

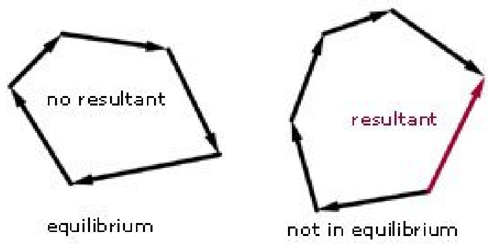
§4.3 Equilibrium of forces

A system is in equilibrium when there is

1. (**translational equilibrium**) no resultant force (net force is zero in any direction): $\sum \vec{\mathbf{F}} = 0$
2. (**rotational equilibrium**) no resultant torque (net torque is zero about any axis of rotation): $\sum \vec{\tau} = 0$

For multiple non-parallel forces, this is illustrated by:

- Forces form a **closed vector polygon**.
- Lines of actions of forces **intersect at one point**, so no resultant moment about point of intersection.



§4.4 Upthrust

Pressure P of a liquid column is

$$P = \rho g h. \tag{24}$$

See derivation

Origin of upthrust: <u>resultant</u> force due to difference in pressure exerted by fluid at top & bottom surfaces of the body.

Archimedes' Principle:

$$\begin{aligned} U &= W_{\text{displaced}} \\ &= \rho_{\text{fluid}} V_{\text{displaced}} \, g \end{aligned} \tag{25}$$

See derivation

For object floating in equilibrium,

$$U = W_{\text{object}}.$$

Derivations

Pressure

$$\boxed{P = \rho g h}$$

Derivation. Given a liquid column of height h and cross-sectional area A, of density rho.

$$m = \rho V = (Ah)\rho$$

Weight W of the liquid column above A is

$$W = mg = \rho V g = Ah\rho g$$

Hence pressure on area A is given by

$$\rho = \frac{F}{A} = \frac{Ah\rho g}{A} = \rho g h$$

□

Upthrust

$$\boxed{U = W_{\text{displaced}}}$$

Derivation. Consider a solid cylinder of height h and cross-sectional area A, submerged in a liquid of density ρ.

Pressure on the top surface is given by

$$p_1 = \rho g h_1 + p_0$$

Hence downward force on top surface is

$$F_1 = (\rho g h_1 + p_0)A$$

Similarly, pressure on the bottom surface is given by

$$p_2 = \rho g h_2 + p_0$$

Upward force on bottom surface is

$$F_2 = (\rho g h_2 + p_0)A$$

Hence, the resultant upward force (upthrust) on the cylinder is

$$\begin{aligned} U &= F_2 - F_1 \\ &= \rho g (h_2 - h_1) A \\ &= \rho g h A \\ &= \rho g V_{\text{displaced}} \\ &= m_{\text{displaced}} g \end{aligned}$$

which is equal to the weight of fluid displaced by the object. □

§5 Work, Energy and Power

§5.1 Work

Work done (by constant force):

$$W = Fs\cos\theta \tag{26}$$

where θ is angle between force F and displacement s.

For variable force,

$$W = \int_{x_i}^{x_f} F\,\mathrm{d}x\,. \tag{27}$$

Work done by gas expanding against constant external pressure:

$$W = p\Delta V. \tag{28}$$

For variable force,

$$W = \int_{V_i}^{V_f} p\,\mathrm{d}V\,.$$

§5.2 Energy conversion and conservation

Principle of conservation of energy:

$$(E_k + E_p)_i + W = (E_k + E_p)_f \tag{29}$$

Remark. Work done by dissipative forces is *negative* as the forces act in opposite direction to displacement.

Gravitational potential energy:

$$\text{GPE} = mgh \tag{30}$$

See derivation

Kinetic energy:

$$\text{KE} = \frac{1}{2}mv^2 = \frac{p^2}{2m} \tag{31}$$

See derivation

Elastic potential energy:

$$\text{EPE} = \frac{1}{2}Fx = \frac{1}{2}kx^2 \tag{32}$$

Work–Energy Theorem:

$$W = \Delta\text{KE} \tag{33}$$

Relationship between conservative force[2] F and potential energy U is

$$F = -\frac{\mathrm{d}U}{\mathrm{d}x}. \tag{34}$$

§5.3 Power

Power:

$$P = \frac{\mathrm{d}W}{\mathrm{d}t} \tag{35}$$

Instantaneous power:

$$P = Fv \tag{36}$$

See derivation

Average power when constant force F acts on object with average velocity v_{avg}:

$$P_{\text{avg}} = Fv_{\text{avg}} \tag{37}$$

§5.4 Efficiency

Efficiency:

$$\eta = \frac{\text{useful power/energy output}}{\text{total power/energy input}} \times 100\% \tag{38}$$

[2]A conservative force is one where work done by the force is independent of its path.

Derivations

Gravitational potential energy

$$\boxed{\text{GPE} = mgh}$$

Derivation. Consider an object being raised from height h_1 to height h_2 by a constant force F equal and opposite to the weight mg of the object (so that object does not gain KE).

$$F = mg$$

Work done by force F changes gravitational potential energy, is given by

$$\begin{aligned} W &= F\Delta h \\ &= F(h_2 - h_1) \\ &= mg(h_2 - h_1) \\ &= mgh_2 - mgh_1 \\ &= \text{GPE}_f - \text{GPE}_i \end{aligned}$$

Therefore, gravitational potential energy is GPE $= mgh$. □

Kinetic energy

$$\boxed{\text{KE} = \frac{1}{2}mv^2}$$

Derivation. Consider a stationary body of mass m which moves a horizontal displacement s under the action of a constant net force F. Since the force is constant, body moves with constant acceleration a.

By N2L,

$$F = ma.$$

The final velocity v of the body is given by

$$\begin{aligned} v^2 &= u^2 + 2as \\ v^2 &= 0^2 + 2as \\ s &= \frac{v^2}{2a} \end{aligned}$$

Hence, work done on the body is

$$W = Fs = ma\left(\frac{v^2}{2a}\right) = \frac{1}{2}mv^2.$$

Work done by force F increases the kinetic energy of the body.

Therefore, the kinetic energy of a body at speed v is KE $= \frac{1}{2}mv^2$. □

Power

$$\boxed{P = Fv}$$

Derivation. For a constant force,

$$P = \frac{\mathrm{d}W}{\mathrm{d}t} = \frac{\mathrm{d}(Fs)}{\mathrm{d}t} = F\frac{\mathrm{d}s}{\mathrm{d}t} = Fv.$$

□

§6 Circular Motion

§6.1 Uniform circular motion

Angular displacement: angle in radians through which a point is rotated.

$$\theta = \frac{s}{r} \tag{39}$$

Angular velocity: rate of change of angular displacement.

$$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t} \tag{40}$$

Relating angular velocity to period and frequency,

$$\omega = \frac{2\pi}{T} = 2\pi f. \tag{41}$$

Linear velocity:

$$v = r\omega \tag{42}$$

Centripetal acceleration:

$$a = \frac{v^2}{r} = r\omega^2 \tag{43}$$

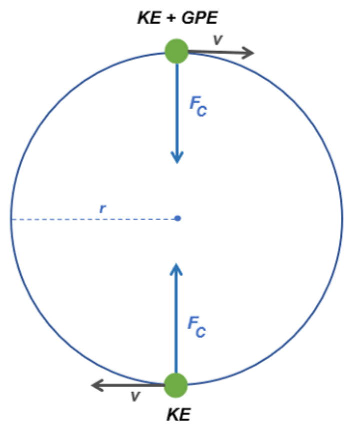
By N2L, **centripetal force**:

$$F_c = \frac{mv^2}{r} = mr\omega^2 \tag{44}$$

Remark. Centripetal force is a **resultant force**; it can be provided by gravitational force, friction force, normal force, etc.

Why does a resultant force exist in a uniform circular motion?
Velocity changes due to change in direction, hence the object undergoes acceleration. By N2L, a resultant force acts on the object.
Since force does not change speed of object, it does no work to accelerate the object, so centripetal force acts perpendicularly to motion, towards centre of circular path.

§6.2 Non-uniform circular motion



Tension in string is max at bottom, min at top.

At the top, string is just taut, $T = 0$ N, thus mg provides centripetal force completely:

$$mg = \frac{m{v_{\text{top}}}^2}{r}$$
$$v_{\text{top}} = \sqrt{gr}$$

At the bottom, by conservation of energy,

$$\frac{1}{2}m{v_{\text{bottom}}}^2 = mg(2r) + \frac{1}{2}m{v_{\text{top}}}^2$$
$$v_{\text{bottom}} = \sqrt{5gr}$$

§7 Gravitational Field

§7.1 Gravitational force

Newton's law of gravitation:

$$\vec{\mathbf{F}} = -\frac{GMm}{r^2}\hat{\mathbf{r}} \tag{45}$$

where $G = 6.67 \times 10^{-11}\ \mathrm{kg^{-1}\,m^3\,s^{-2}}$ is gravitational constant.

Remark. Negative sign due to attractive nature of gravitational force.

Remark. Gravitational forces between two masses are action-reaction pair.

Remark. **Point masses** have their masses concentrated at one point. Two objects can be considered point masses when they are placed sufficiently far apart such that their dimensions become negligible compared to separation.

§7.2 Gravitational field

Field of force: region of space where non-contact force acts on object in the field

- Field lines indicate direction of field of force. Density of field lines corresponds to strength of field.
- Field lines directed radially towards centre of Earth, gravitational field around Earth is non-uniform (field strength stronger near Earth, weaker further away from Earth).

  Near Earth surface, field lines parallel and equally spaces, so gravitational field is uniform, $g \approx 9.81\ \mathrm{N\,kg^{-1}}$ (acceleration of free fall).

Gravitational field strength:

$$\vec{\mathbf{g}} = -\frac{GM}{r^2}\hat{\mathbf{r}} \tag{46}$$

See derivation

Neutral point: resultant gravitational field strength = 0

§7.3 Gravitational potential energy

Gravitational potential energy:

$$U = -\frac{GMm}{r} \tag{47}$$

Remark. Negative sign cannot be omitted. U is negative:

- U is defined to be 0 at $r = \infty$
- gravitational force is attractive
- work done is negative as force and displacement act in opposite directions

Gravitational potential:

$$\phi = -\frac{GM}{r} \tag{48}$$

Remark. For the same reasons as above, ϕ is negative.

		At a point distant r from M	For an object (m)	
Vectors	$\propto \frac{1}{r^2}$	Gravitational Field Strength $\vec{g} = -\frac{GM}{r^2}\hat{r}$	Gravitational Force $\vec{F} = -\frac{GMm}{r^2}\hat{r}$	$\vec{F} = m\vec{g}$
Scalars	$\propto \frac{1}{r}$	Gravitational Potential $\phi = -\frac{GM}{r}$	Gravitational Potential Energy $U = -\frac{GMm}{r}$	$U = m\phi$
		$\vec{g} = -\frac{d\phi}{dr}\hat{r}$	$\vec{F} = -\frac{dU}{dr}\hat{r}$	

Relationship between $\vec{g}$ and ϕ

When the equation $\vec{F} = -\frac{dU}{dr}\hat{r}$ is divided by m, i.e.

$\vec{F} = -\frac{dU}{dr}\hat{r}$

$\rightarrow\ m\vec{g} = -\frac{d(m\phi)}{dr}\hat{r}$

$\rightarrow\ \vec{g} = -\frac{d\phi}{dr}\hat{r}$

Relationship between $\vec{F}$ and U

Recall that in any field of force, when an object in that field is moved through a small distance dr, its potential energy would be changed by dU. The force (due to the field) acting on the object placed at that region, $\vec{F}$, would be expressed as $\vec{F} = -\frac{dU}{dr}\hat{r}$.

Note:
Both sides of the equation are **vectors**, and the negative sign indicates that they are in opposite directions, i.e., the direction of gravitational force $\vec{F}$ acts in the direction of **decreasing** U.

§7.4 Applications

§7.4.1 Orbit velocity

For satellite of mass m orbiting planet of mass M, gravitational force provides centripetal force:

$$F_g = F_c$$
$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$
$$\boxed{v = \sqrt{\frac{GM}{r}}}$$

§7.4.2 Kinetic energy

For satellite in orbit, gravitational force provides centripetal force:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$
$$mv^2 = \frac{GMm}{r}$$
$$\frac{1}{2}mv^2 = \frac{GMm}{2r}$$
$$\boxed{\mathrm{KE} = \frac{GMm}{2r}}$$

§7.4.3 Kepler's third law

Gravitational force provides centripetal force:

$$\frac{GMm}{r^2} = mr\omega^2 = mr\left(\frac{2\pi}{T}\right)^2$$
$$T^2 = \frac{4\pi^2}{GM}r^3$$
$$\boxed{T^2 \propto r^3}$$

§7.4.4 Escape speed

Escape speed: minimum speed required to escape effect of gravitational field.

Apply conservation of energy. At infinity, $U_f = 0$ (by definition of gravitational potential energy) and KE = 0 (by definition of escape speed).

$$\mathrm{KE}_i + U_i = \mathrm{KE}_f + U_f$$
$$\frac{1}{2}mv^2 + \left(-\frac{GMm}{r}\right) = 0$$
$$\boxed{v = \sqrt{\frac{2GM}{r}}}$$

§7.4.5 Geostationary satellite

Characteristics:

1. Orbital period is the same as the rotational period of Earth about its axis, i.e. $T = 24$ hr.
2. Moves in the same direction as the rotation of Earth about its own axis, i.e. from west to east.
3. Vertically above the equator, so that its axis of rotation is the same as the Earth.
   - Gravitational force is the resultant force that provides centripetal force for satellite.
   - Gravitational force is directed towards centre of Earth, centripetal force is directed towards centre of orbit,
   - so centre of orbit must be centre of Earth.

Derivations

Gravitational field strength

$$\boxed{g = -\frac{GM}{r^2}}$$

Derivation. Newton's Law of Gravitation states that

$$F = -\frac{GMm}{r^2}$$

By the definition of gravitational field strength,

$$g = \frac{F}{m} = \frac{-\frac{GMm}{r^2}}{m} = -\frac{GM}{r^2}$$

□

Part III

Thermal Physics

§8 Temperature and Ideal Gases

§8.1 Thermal equilibrium

Temperature: measure of degree of hotness of. Thermal energy moves from object at higher temperature to object at lower temperature.

Remark. Temperature does not measure amount of thermal energy; it only indicates direction of heat flow.

Heat: (thermal) energy that flows from region of higher to lower temperature[3].

§8.2 Temperature scales

§8.2.1 Empirical scale

Empirical temperature scale: scale of temperature based on variation of some physical property with temperature.

1. Choose appropriate thermometric property: varies linearly with temperature
   - volume of fixed mass of liquid (liquid-in-glass thermometer)
   - resistance of metal (platinum resistance thermometer)
   - pressure of fixed mass of gas at constant volume (constant volume gas thermometer)
   - e.m.f. between junctions of dissimilar metals at different temperatures (thermocouple thermometer)
2. Select two fixed points

   Usually **ice point** of water (0 °C) and **steam point** of water (100 °C).
3. Calibrate thermometer

   Place it in systems of lower and upper fixed points. Record values of thermometric quantity. Assume linear relationship between these two points.

X_θ denotes value of thermometric property at temperature θ, then

$$\frac{\theta}{100} = \frac{X_\theta - X_i}{X_s - X_i} \tag{49}$$

where X_i is value of X at ice point, X_s is value of X at steam point.

Remark. Use ratio to solve problems.

Remark. Assumption of linearity of thermometric properties may be wrong or inaccurate; instead, actual behaviour of thermometric property is non-linear. Hence empirical scales are always slightly wrong, *except at the fixed points.*

§8.2.2 Thermodynamic scale

Absolute zero: temperature at which all substances have minimum internal energy.

Thermodynamic temperature scale: does not depend on thermometric property of any particular substance, has fixed points at absolute zero and triple point of water[4].

To convert temperatures measured in °C to K,

$$T/\mathrm{K} = T/{}^\circ\mathrm{C} + 273.15.$$

Remark. NOT 273!

[3] through conduction, convection and radiation.

[4] The particular temperature and pressure (273.16 K, 4.58 mmHg) at which the three states of water (solid, liquid, vapour) can co-exist at equilibrium, i.e. transition state curves meet.

§8.3 Equation of state

For a fixed amount of gas, the following relationships can be deduced experimentally.

- **Charles' Law**: $V \propto T$ at constant p
- **Boyle's Law**: $p \propto \frac{1}{V}$ for constant T
- **Gay–Lussac's Law**: $p \propto T$ for constant V

Combining the above gives **equation of state** of ideal gas, in *moles*:

$$pV = nRT \tag{50}$$

where $R = 8.31\ \mathrm{J\,mol^{-1}\,K^{-1}}$ is molar gas constant.

One **mole** of any substance contains 6.02×10^{23} particles.

Avogardo constant: number of particles in one mole.
$N_A = 6.02 \times 10^{23}\ \mathrm{mol^{-1}}$

Boltzmann's constant: gas constant per molecule.
$k = 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}}$

Rewriting eq. (50) in *number of particles* gives

$$pV = NkT \tag{51}$$

Molar mass: mass of one mole.

$$M_r = \frac{m_{\text{gas}}}{n}$$

Molar volume: volume of one mole.

$$V_m = \frac{V}{n}$$

§8.4 Kinetic theory of gases

Assumptions of kinetic theory of gases:

- Gas consists of a very large number of particles.
- Gas particles are in constant random motion and obey Newton's laws of motion.
- No forces of attraction or repulsion between gas particles except during collision.
- Gas particles behave as perfectly elastic, identical, hard spheres,
- Total volume of the gas particles is negligible compared to volume of container.
- Time duration of collision is negligible compared to time interval between collisions.

Explain how molecular movement causes pressure exerted by gas.

1. When gas particles collide with walls of container, change in momentum of gas particles.
2. By N2L, walls exert a force on the particles.
3. By N3L, a force of equal magnitude exerted by gas particles on walls of container.
4. Since $P = \frac{F}{A}$, pressure exerted on walls of container.

We have

$$pV = \frac{1}{3}Nm\langle c^2 \rangle \tag{52}$$

See derivation

Related equations include

$$pV = \frac{1}{3}Nm\langle c^2 \rangle$$
$$pV = \frac{1}{3}M\langle c^2 \rangle$$
$$pV = \frac{1}{3}nM_r\langle c^2 \rangle$$

Equating to eq. (51) gives mean translational kinetic energy $\langle \mathrm{KE} \rangle$ of one molecule:

$$\frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT \tag{53}$$

Thus mean KE of a molecule of ideal gas is *proportional* to thermodynamic temperature.

Hence total translational kinetic energy of N particles is

$$\mathrm{KE} = N\langle \mathrm{KE} \rangle = \frac{3}{2}NkT = \frac{3}{2}nRT$$

Derivations

Kinetic theory of gas

$$\boxed{pV = \frac{1}{3}Nm\langle c^2\rangle}$$

Derivation. Consider ideal gas consisting of N identical particles, each of mass m, in cubical container of side length L.

Since particles move randomly, no preferred direction of travel along the x-, y- and z-axes. We expect one-third of N particles move along each axis.

1. **Change in momentum**

   Consider one-dimensional case along x-axis. One gas molecule of mass m collides elastically with wall with velocity c_x, leaves wall with velocity $-c_x$. Change in momentum of gas molecule is

   $$\Delta p = -2mc_x.$$

   By conservation of linear momentum, wall experiences change in momentum of

   $$\Delta p_{\text{wall}} = 2mc_x.$$

2. **Force by N2L**

   After the collision, this molecule travels distance of $2L$ back and forth, collides with the same wall. Time interval between successive collisions is

   $$\Delta t = \frac{2L}{c_x}.$$

   Rate of change of momentum of wall due to one molecule is

   $$\frac{\Delta p_{\text{wall}}}{\Delta t} = -\frac{m{c_x}^2}{L}.$$

   By Newton's 2nd law, net force on wall is rate of change of momentum of wall due to all N particles:

   $$\begin{aligned} F_{\text{wall}} &= \frac{m{c_{x1}}^2}{L} + \cdots + \frac{m{c_{xN}}^2}{L} \\ &= \frac{m}{L}\left({c_{x1}}^2 + \cdots + {c_{xN}}^2\right) \\ &= \frac{Nm}{L}\langle {c_x}^2\rangle \end{aligned}$$

   where mean square speed $\langle {c_x}^2\rangle = \dfrac{{c_{x1}}^2 + \cdots + {c_{xN}}^2}{N}$.

3. **Pressure**

   Since area of the wall is L^2, pressure on wall is

   $$p = \frac{F}{L^2} = \frac{Nm}{V}\langle {c_x}^2\rangle \tag{1}$$

   since $V = L^3$.

   Applying Pythagoras' Theorem to 3D velocity vector of molecule,

   $$c^2 = {c_x}^2 + {c_y}^2 + {c_z}^2 \implies \langle c^2\rangle = \langle {c_x}^2\rangle + \langle {c_y}^2\rangle + \langle {c_z}^2\rangle$$

   Since there are $\dfrac{N}{3}$ particles moving along each axis, $\langle {c_x}^2\rangle = \langle {c_y}^2\rangle = \langle {c_z}^2\rangle$. Hence

   $$\langle {c_x}^2\rangle = \frac{1}{3}\langle c^2\rangle$$

   Substituting this into (1) gives

   $$pV = \frac{1}{3}Nm\langle c^2\rangle.$$

$\square$

§9 First Law of Thermodynamics

§9.1 Specific heat capacity, latent heat

Specific heat capacity:

$$Q = mc\Delta T \tag{54}$$

Specific latent heat:

$$Q = mL \tag{55}$$

Phase change: transition from one state of matter to another. During phase change, latent heat is given off or absorbed, temperature does not change.

§9.2 Internal energy

The **state**[5] of a system is defined by pressure p, volume V, thermodynamic temperature T.

Internal energy:

$$U = \text{KE}_{\text{random}} + \text{PE}_{\text{random}} \tag{56}$$

For ideal gas, no intermolecular attraction so $\text{PE}_{\text{random}} = 0$:

$$U = \text{KE}_{\text{random}} = N\langle\text{KE}\rangle = \frac{3}{2}NkT$$

This means as temperature of ideal gas increases, mean random KE of molecules increases, sum of random KE increases, so internal energy increases.

§9.3 First law of thermodynamics

Work done W is area under pressure-volume graph:

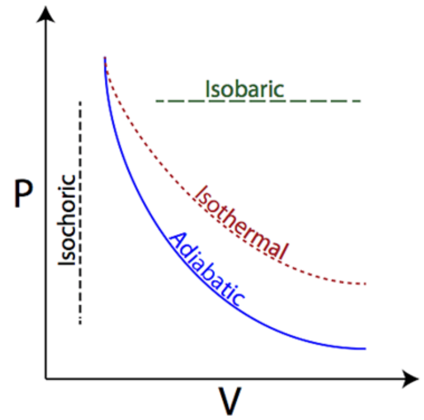
$$W = \int_{V_i}^{V_f} p\,\mathrm{d}V\,.$$

First law of thermodynamics:

$$\Delta U = Q_{\text{to}} + W_{\text{on}} \tag{57}$$

where ΔU is change in internal energy, Q_{to} is heat supplied to system, W_{on} is work done on system.

The **state** of a system can be represented by a point on $p-V$ graph.



Thermodynamic processes:

- **Isothermal**: constant temperature

  $\Delta U = 0$, change in state occurs along an isotherm

- **Isobaric**: constant pressure

  Work done by system $W_{\text{by}} = -W_{\text{on}} = p\Delta V$

- **Isochoric**: constant volume

  $W = 0 \implies \Delta U = Q$

- **Adiabatic**: no heat exchange between system and environment

  $Q = 0$. Since no heat supplied to system, system uses thermal energy to do work to push back environment, so temperature decreases.

  Paths for adiabatic changes are steeper than isotherms.

Cyclic process: system starts and ends with same state, so no change in internal energy ($\Delta U = 0$)

[5] not solid, liquid or gas – these are *phases*. The concept of state must be clear!

Part IV
Oscillation and Waves

§10 Oscillations

Frequency:

$$f = \frac{1}{T} \tag{58}$$

Angular frequency:

$$\omega = 2\pi f = \frac{2\pi}{T} \tag{59}$$

§10.1 Kinematics and energy

Displacement-time:

When the body is at equilibrium position at $t = 0$,

$$x = x_0 \sin \omega t. \tag{60}$$

When the body is at extreme position at $t = 0$,

$$x = x_0 \cos \omega t.$$

Velocity-time:

$$v = \frac{\mathrm{d}x}{\mathrm{d}t} = v_0 \cos \omega t \tag{61}$$

Acceleration-time:

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = -a_0 \sin \omega t \tag{62}$$

Velocity-displacement:

$$v = \pm\omega\sqrt{{x_0}^2 - x^2} \tag{63}$$

Acceleration-displacement:[6]

$$a = -\omega^2 x \tag{64}$$

Phase difference

- Graph of $x = x_0 \cos(\omega t + \phi)$ is displaced to the left. Motion *leads* by time $\frac{\phi}{\omega}$.
- Graph of $x = x_0 \cos(\omega t - \phi)$ is displaced to the right. Motion *lags* by time $\frac{\phi}{\omega}$.

For oscillator in SHM, total energy is sum of KE and PE:

$$E = E_k + E_p.$$

By $E_k = \frac{1}{2}mv^2$,

$$\begin{aligned} E_k &= \frac{1}{2}m\omega^2{x_0}^2\cos^2\omega t \\ &= \frac{1}{2}m\omega^2\left({x_0}^2 - x^2\right). \end{aligned} \tag{65}$$

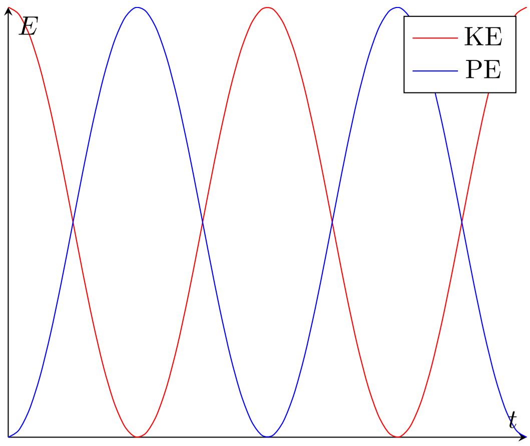
Then

$$E = E_{k,max} = \frac{1}{2}m\omega^2{x_0}^2. \tag{66}$$

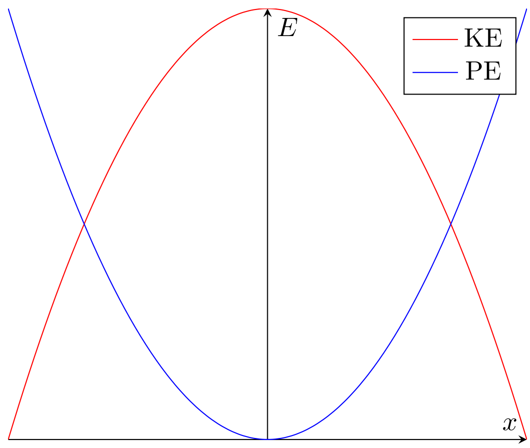
Thus

$$\begin{aligned} E_p &= \frac{1}{2}m\omega^2{x_0}^2\sin^2\omega t \\ &= \frac{1}{2}m\omega^2x^2. \end{aligned} \tag{67}$$

Energy-time graph (for one period):



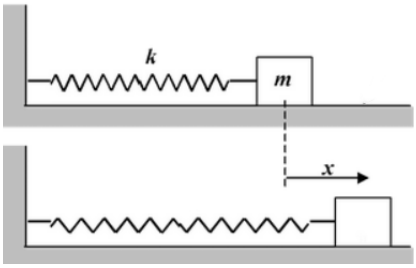
Energy-displacement graph (for one period):



[6] defining equation of simple harmonic motion

§10.2 Simple harmonic motion

Spring-mass system:



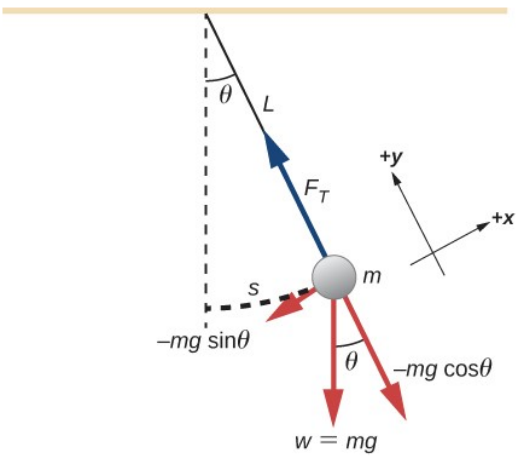
Restoring force is $F = -kx$. By N2L,

$$\sum F = ma = -kx$$
$$a = -\frac{k}{m}x$$

Comparing with $a = -\omega^2 x$,

$$\omega = \sqrt{\frac{k}{m}}.$$

Simple pendulum:



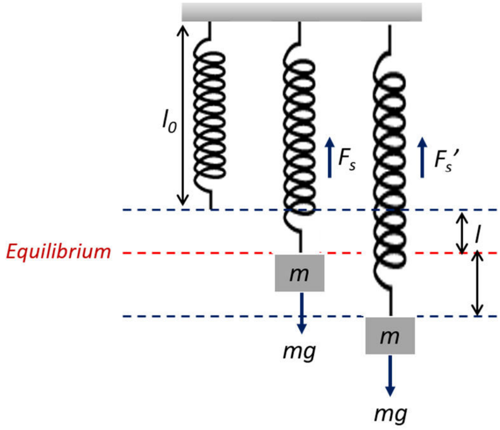
Restoring force is component of weight tangential to circumference of its swing: $mg\sin\theta$. For small θ, by small angle approximation, $\sin\theta \approx \theta \approx \frac{x}{l}$. By N2L,

$$\sum F = ma = -\frac{mgx}{l}$$
$$a = -\frac{g}{l}x$$

Comparing with $a = -\omega^2 x$,

$$\omega = \sqrt{\frac{g}{l}}.$$

Vertical spring-mass system:



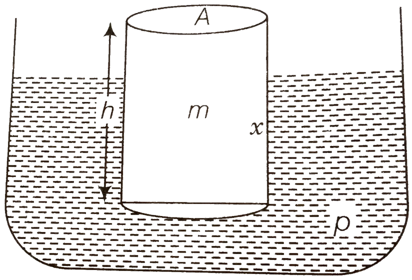
At equilibrium, spring force balances weight: $ke = mg$. At lowest point, by N2L,

$$\sum F = ma = mg - k(e + x)$$
$$a = -\frac{k}{m}x$$

Comparing with $a = -\omega^2 x$,

$$\omega^2 = \frac{k}{m}.$$

Floating block:



Restoring force is difference between upthrust and weight. By N2L,

$$\sum F = ma = mg - \rho(A(h + x))g$$
$$= -\rho(Ax)g$$
$$a = -\frac{\rho Ag}{m}x$$

Comparing with $a = -\omega^2 x$,

$$\omega = \sqrt{\frac{\rho Ag}{m}}.$$

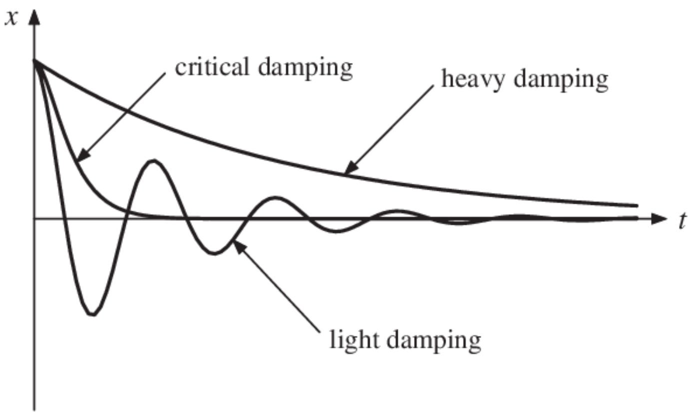
§10.3 Resonance and damping

Degrees of damping:

1. **Light damping**: continues to oscillate, amplitude decreases gradually with time but period remains almost the same

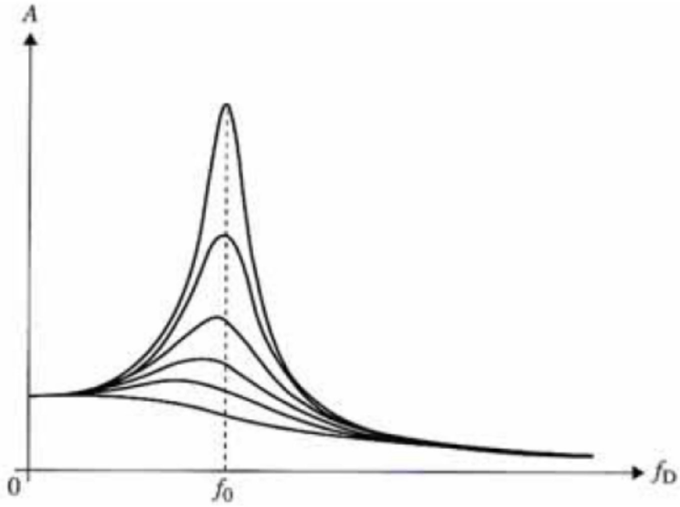
2. **Heavy damping**: does not oscillate, takes a long time to return to equilibrium
e.g. door damper

3. **Critical damping**: does not oscillate, returns to equilibrium in the shortest possible time
e.g. damping system of car



Forced oscillation: system oscillates at the frequency of external periodic force.

Resonance: $f_{\text{driving}} = f_0$



Amplitude of forced oscillation changes with driving frequency. When $f = f_0$, resonance occurs, amplitude is maximum.

Effect of increased damping on resonance curve:

- Lower at all frequencies
- Flatter peak
- Peak shifts to the left slightly

There are some circumstances in which resonance is useful, such as in magnetic resonance imaging (MRI), and other circumstances in which resonance should be avoided, such as in bridge design to prevent collapse due to resonant oscillations.

§11 Wave Motion

§11.1 Terminology

Graphical representations

- **Displacement-distance graph**: displacement of particles at a particular instant in time.

  Determine wavelength, amplitude

- **Displacement-time graph**: displacement of a single particle varies with time.

  Determine period, amplitude

- **Pressure-distance graph**: for longitudinal waves

§11.1.1 Phase, phase difference

Phase: angle which indicates fraction of a cycle completed by particle / wave.

Phase difference $\Delta\phi$: angle which indicates how much one wave / particle is *out of step* with another.

- **In phase**: in step with one another ($\Delta\phi = 0$)
- **Out of phase**: not in step with one another ($\Delta\phi \neq 0$)
- **Anti-phase**: out of phase by half cycle ($\Delta\phi = \pi$)

For displacement-distance graph,

$$\Delta\phi = \frac{\Delta x}{\lambda} \times 2\pi \tag{68}$$

For displacement-time graph,

$$\Delta\phi = \frac{\Delta t}{T} \times 2\pi \tag{69}$$

§11.1.2 Energy, intensity

From SHM, energy associated with oscillation is proportional to square of amplitude:

$$E = \frac{1}{2}m\omega^2{x_0}^2 = \frac{1}{2}m(2\pi f)^2{x_0}^2 \implies \boxed{E \propto f^2A^2}$$

Intensity:

$$I = \frac{P}{\text{Area}} \tag{70}$$

At constant f,

$$I \propto E \text{ and } E \propto A^2 \implies \boxed{I \propto A^2}$$

For three-dimensional transmission, energy emitted radially outwards onto spherical surface (surface area of $4\pi r^2$):

$$I \propto \frac{1}{r^2}$$

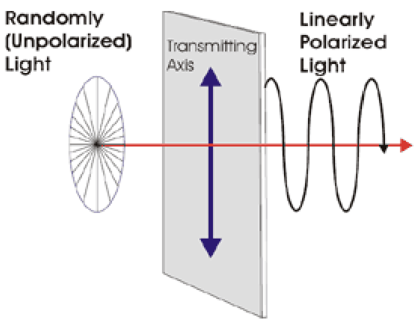
For two-dimensional transmission, energy emitted radially outwards onto cylindrical surface (surface area of $2\pi rh$):

$$I \propto \frac{1}{r}$$

§11.2 Polarisation

Remark. Polarisation is associated only with *transverse waves*; longitudinal waves (e.g. sound waves) cannot be polarised, as they do not have oscillations in the plane normal to direction of transfer of energy of wave.

Polariser: only allows waves whose direction of oscillation is parallel to transmission axis to pass through.



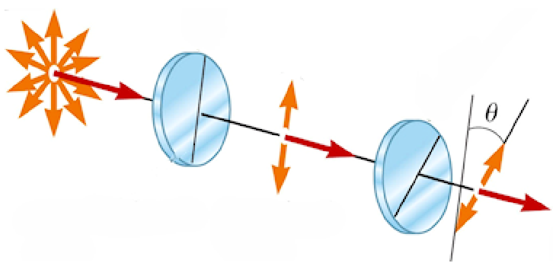
§11.2.1 Unpolarised wave

When unpolarised wave is incident on a polariser, intensity is halved[7], amplitude unchanged.

$$I = \frac{1}{2}I_0 \tag{71}$$

§11.2.2 Polarised wave

Polariser only allows *component* of oscillation *parallel* to transmission axis to pass through.



[7] unpolarised wave is a random mixture of all states of polarisation, vertical and horizontal components are equal on average.

Let I_0 denote intensity of polarised light, I denote intensity after passing through analyser, θ denote angle between transmission axes of polariser and analyser. Then

$$I_0 \propto {A_0}^2 \tag{1}$$

Component of A_0 parallel to transmission axis of analyser is $A = A_0 \cos\theta$,so

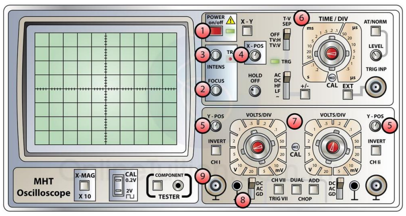
$$I \propto {A_0}^2 \cos^2\theta \tag{2}$$

Dividing (2) by (1) and rearranging gives **Malus' Law**:

$$I = I_0 \cos^2\theta \tag{72}$$

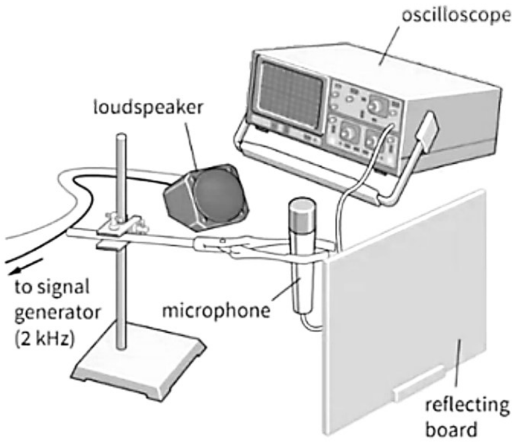
Remark. I max when $\theta = 0°$; I min when $\theta = 90°$.

§11.3 Cathode ray oscilloscope (c.r.o.)



Determine frequency of sound wave

1. Signal fed through microphone into c.r.o.
2. Turn on time-base of c.r.o., trace on screen displays displacement against time.
3. Adjust time-base until stationary trace obtained.
4. Find period T, calculate frequency f using $f = \dfrac{1}{T}$.



Determine wavelength of sound wave

1. Loudspeaker delivers sound via signal generator. Incident wave is directed towards and is reflected at reflecting board.
2. Superposition of incident wave and reflected wave produces stationary wave.
3. Connect microphone to c.r.o. (time-base switched off[8]). Move it along the line between loudspeaker and reflecting board.
4. When microphone is at positions of minimum amplitude (nodes), signal displayed is minimum; at positions of maximum amplitude (antinodes), signal displayed is maximum.
5. Measure distance across several nodes, calculate average distance d between two adjacent nodes.
6. Separation of two adjacent nodes is equal to half a wavelength: $d = \dfrac{1}{2}\lambda$.

Derivations

Wave speed

$$\boxed{v = f\lambda}$$

Derivation. One wavelength λ is distance travelled by wave during one complete oscillation of the source.

Distance travelled by wave during n complete oscillations is $n\lambda$.

Speed of wave is distance per unit time, $v = \dfrac{n\lambda}{t}$.

Frequency is number of oscillations per unit time, $f = \dfrac{n}{t}$.

Hence

$$f\lambda = \left(\frac{n}{t}\right)\lambda = \frac{n\lambda}{t} = v.$$

□

[8] so that the spot does not move across the screen. The spot moves up and down the screen, and the height of the vertical trace gives a measure of the amplitude of the sound

§12 Superposition

When two waves meet in opposite directions, they superpose to form stationary wave.

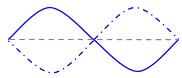
When two waves meet in same direction, they interfere constructively / destructively.

§12.1 Superposition

Distance between two adjacent nodes / antinodes:

$$d = \frac{1}{2}\lambda$$

Drawing of stationary wave: (draw the two extremes)



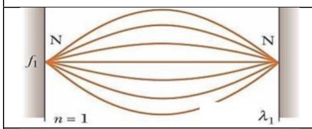
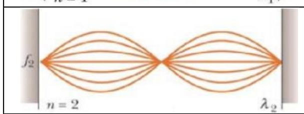
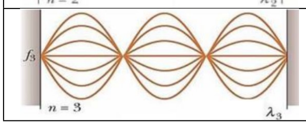
Progressive vs stationary waves

	Progressive wave	**Stationary wave**
Amplitude	Same for all particles in wave motion	Vary from zero (node) to max (antinode)
Frequency	Particles vibrate in SHM with frequency of progressive wave	Particles vibrate in SHM with frequency of stationary wave
Wavelength	Shortest distance between two points in phase	Twice the distance between a pair of adjacent nodes / antinodes
Phase	All particles within one wavelength have different phases	All particles within a loop vibrate in phase, paricles in adjacent loops are antiphase
Wave profile	Wave profile advances with speed of wave	Wave profile does not advance
Energy	Transported in the direction of travel of wave	Stored within vibrations of stationary wave

§12.1.1 Transverse stationary waves

Stretched string:

- **Experiment set-up**:
  String of length L stretched tightly between fixed supports. When plucked, progressive wave produces, reflected at fixed ends, travel backwards. Incident and reflected waves superpose to form a stationary wave.
- **Node**: fixed ends

Mode of vibration	**Wavelength**	**Frequency**	**Harmonic**	**Overtone**
	$\lambda_1 = 2L$	$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$	1st	-
	$\lambda_2 = L$	$f_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2f_1$	2nd	1st
	$\lambda_3 = \frac{2}{3}L$	$f_3 = \frac{v}{\lambda_3} = \frac{3v}{2L} = 3f_1$	3rd	2nd

Integer number of lops must fit exactly into the length of the string:

$$L = n\left(\frac{\lambda}{2}\right)$$

n-th harmonic:

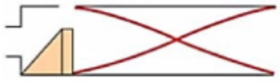
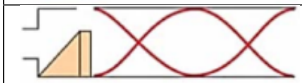
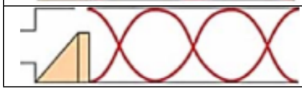
$$f_n = n\left(\frac{v}{2L}\right)$$

§12.1.2 Longitudinal stationary waves

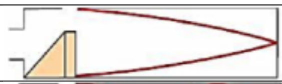
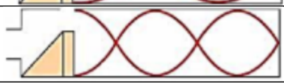
Open/closed pipe:

- **Experiment set-up**:
  Sound wave enters pipe via open end, travels from open end towards closed end, reflected when it hits wall of closed end. Incident and reflected waves superpose to form a stationary wave.
- **Node**: closed end (air particles are not moving, powder collect at nodes)
  Antinode: open end (air particles are moving with max amplitude, powder pushed away from antinodes to nodes)

Open pipe:

Wave profile	Wavelength	Frequency	Harmonic	Overtone
	$\lambda_1 = 2L$	$f_1 = \dfrac{v}{\lambda_1} = \dfrac{v}{2L}$	1st	–
	$\lambda_2 = L$	$f_2 = \dfrac{v}{\lambda_2} = \dfrac{v}{L} = 2f_1$	2nd	1st
	$\lambda_3 = \dfrac{2}{3}L$	$f_3 = \dfrac{v}{\lambda_3} = \dfrac{3v}{2L} = 3f_1$	3rd	2nd

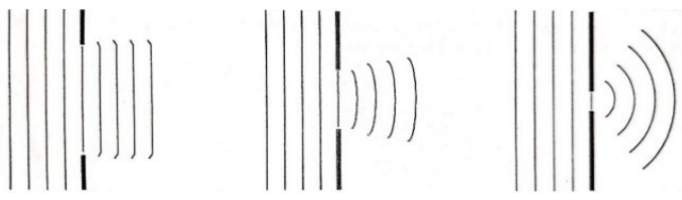
Closed pipe:

Wave profile	Wavelength	Frequency	Harmonic	Overtone
	$\lambda_1 = 4L$	$f_1 = \dfrac{v}{\lambda_1} = \dfrac{v}{4L}$	1st	–
	$\lambda_2 = \dfrac{4}{3}L$	$f_2 = \dfrac{v}{\lambda_2} = \dfrac{3v}{4L} = 3f_1$	3rd	1st
	$\lambda_3 = \dfrac{4}{5}L$	$f_3 = \dfrac{v}{\lambda_3} = \dfrac{5v}{4L} = 5f_1$	5th	2nd

In practice, antinode at open end occurs *slightly outside* the pipe, so **end correction** is needed.

§12.2 Interference

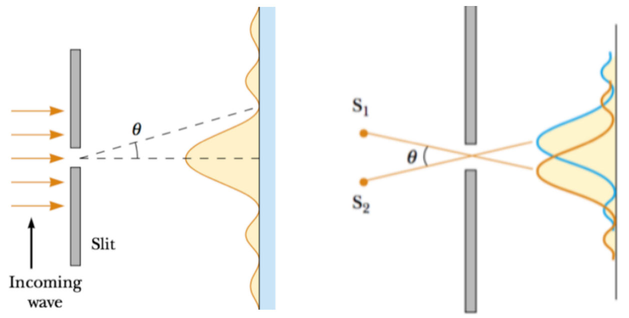
For significant **diffraction**, size of slit comparable to wavelength (same order), i.e. $b \approx \lambda$



Construtive difference: path difference is a whole number of wavelengths, i.e. $n\lambda$.

Destrutive difference: path difference is an odd number of half wavelengths, i.e. $(n + \frac{1}{2})\lambda$

§12.2.1 Single slit diffraction



Positions of m-th minimum:

$$b \sin\theta = m\lambda \tag{73}$$

where b is slit width.

Resolution of two objects: ability to see as distinct two objects that are distinct.

By **Rayleigh criterion**, minimum angle for two sources to be resolved is

$$\theta_{\text{min}} \approx \frac{\lambda}{b}. \tag{74}$$

§12.2.2 Young's double slit diffraction

For two-source interference fringes to be *observable*,

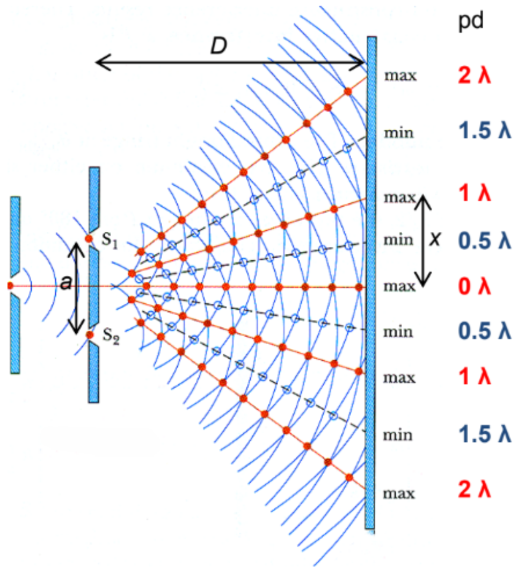
1. Waves must <u>meet</u>
2. Waves must be <u>coherent</u>
3. Waves have (approximately) equal amplitudes
4. Transverse waves must be either <u>unpolarised</u> or <u>polarised in the same plane</u>
5. Split separation is of <u>same order</u> as wavelength, i.e $b \approx \lambda$

When waves of amplitude A_1 and A_2 interfere,

$$\frac{I_{\text{dark fringe}}}{I_{\text{bright fringe}}} = \left(\frac{A_{\text{dark fringe}}}{A_{\text{bright fringe}}}\right)^2 = \left(\frac{|A_1 - A_2|}{A_1 + A_2}\right)^2$$

since $I \propto A^2$.

Remark. When amplitude of one wave is smaller/larger than the other wave, intensity of bright fringes decrease & intensity of dark fringes increase due to incomplete destructive interference.

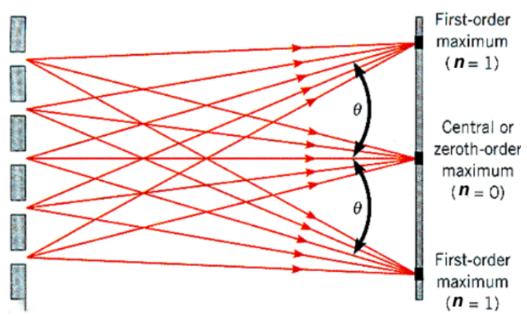


If $a \ll D$, alternating regions of bright and dark fringes are observed.[9]

$$\lambda = \frac{ax}{D} \tag{75}$$

where x is fringe separation, a is slit separation, D is slit-screen distance.

§12.2.3 Diffraction grating



For the n-th order maximum,

$$d \sin\theta = n\lambda \tag{76}$$

where split separation $d = \dfrac{1}{\text{number of lines per unit length}}$.

Remark. Do not use small angle approximation, as θ is not small.

[9] slit separation a should be much smaller than the slit to screen distance D, so that the small angle approximation $\tan\theta \approx \sin\theta$ used in the derivation of Young's double slit formula is valid.

Part V

Electricity and Magnetism

§13 Electric Fields

Types of particles:

Particle	Charge
proton	$+e$
electron	$-e$
α-particle	$+2e$

§13.1 Terminology

Coulomb's law:

$$\vec{\mathbf{F}} = \frac{Qq}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}} \tag{77}$$

where $\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F\,m^{-1}}$ is permittivity of free space (can be taken to be equal to that of air unless specified otherwise).

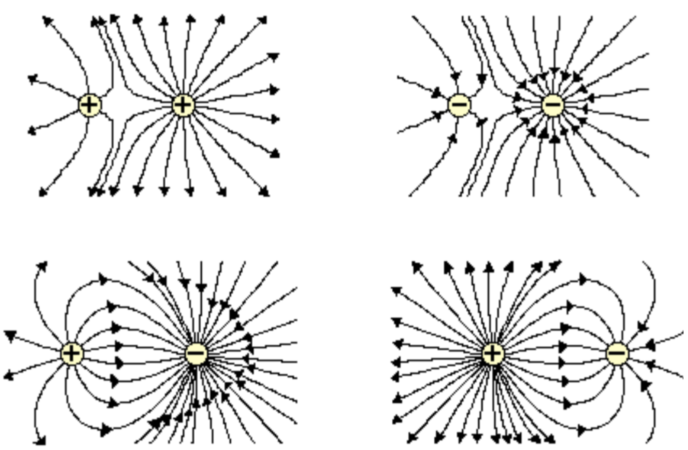
Remark. Electric force is repulsive when $Qq > 0$, attractive when $Qq < 0$.

Remark. In terms of order of magnitude, electric force in order of $\frac{(10^{-19})^2}{10^{-12}} = 10^{-26}$ is about 30 orders larger than gravitational force in order of $10^{-11}(10^{-27})^2 = 10^{-65}$.

Representation of electric field using **field lines**:

- Indicates direction of force on a positive charge if it is placed at that point in the field.
- Density of field lines is proportional to electric field strength.
- Electric field lines are directed away from positive to negative charges, never intersect each other, never created or annihilated in vacuum.

For positive charge, electric field lines are directed radially from centre of charge.



Electric field strength:

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}} \tag{78}$$

Remark. We take charge q to be infinitesimally small so that the field it generates does not disturb that of the "source charge", i.e. charge Q.

Remark. E inside conductor is zero, V inside conductor equals to that on the surface (within conductor, distribute equipotential lines).

Comparison between electric field and gravitational field:

- Qualitative aspect: Gravitational force results from interaction between masses; electric force results from interaction between charges.
- Quantative aspect: Both fields are inverse square law fields.

Electric potential:

$$V = \frac{W}{q} = \frac{Q}{4\pi\varepsilon_0 r} \tag{79}$$

where W is the work done on the charge.

For positive charge, V decreases with distance from charge; for negative charge, V increaases with distance from charge.

Remark. Inside conductor, V equals to that on surface of conductor.

Point charges Q and q at distance r apart, **electric potential energy** of the system of two charges is

$$U = qV = \frac{Qq}{4\pi\varepsilon_0 r}. \tag{80}$$

Remark. EPE can be negative, if one charge is positive and the other is negative.

For a conservative force,

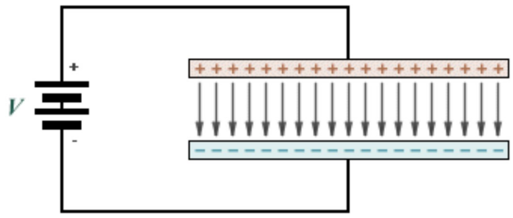
$$F = -\frac{\mathrm{d}U}{\mathrm{d}r} \tag{81}$$

$$E = -\frac{\mathrm{d}V}{\mathrm{d}r} \tag{82}$$

Remark. Graphically, field strength at a point is numerically equal to potential gradient at that point.

Remark. Negative sign means potential decreases in the direction of field lines.

§13.2 Parallel plates



Uniform electric field set up, so electric field strength E is constant: $F = qE$.

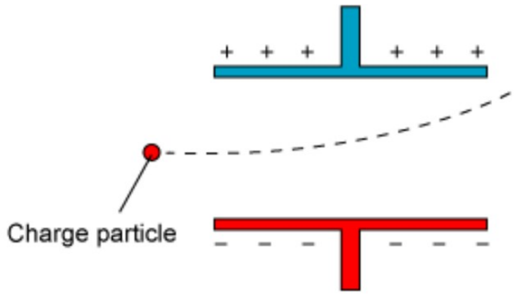
$$E = \frac{|\Delta V|}{d} \tag{83}$$

where $|\Delta V|$ is p.d. across plates, d is separation of plates.

Positive charges move from places of high potential to lower potential, EPE increases.

Negative charges move from places of low potential to higher potential, EPE decreases.

§13.2.1 Parabolic motion



By N2L,

$$a = \frac{F}{m} = \frac{qE}{m} = \frac{q|\Delta V|}{md}$$

which is constant.

§13.2.2 Millikan's oil drop

Charged droplet balanced in between two horizontal plates. Electric force acts upwards, weight acts downwards. By N2L,

$$F_E = F_g \implies \frac{qV}{d} = mg.$$

§14 Current of Electricity

§14.1 Electric current

Elementary charge: $e = 1.6 \times 10^{-19}$ C

For a steady current,

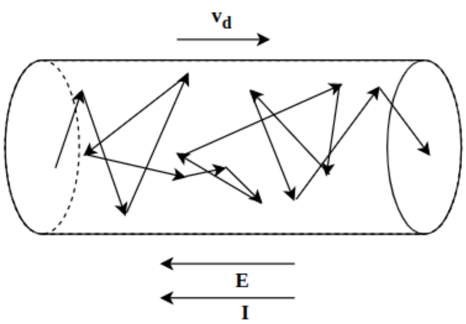
$$Q = It. \tag{84}$$

Remark. By convention, direction of current is direction that positive charges move. Still, note that current is due to flow of electrons; lattice atoms/ions do not move.

Transport equation:

$$I = nAvq \tag{85}$$

where v is **drift velocity**.



Remark. Charge carriers experience electrical force in all directions due to collisions with lattice ions, produce a range of velocities. Drift velocity is an average, so drift velocity is much lower than velocity that charge carriers could achieve from potential difference applied to wire.

Remark. When a domestic lighting circuit is switched on, all electrons in the wite and filament start to move together, so the lights come on almost immediately.

§14.2 Potential difference and electromotive force

Potential difference:

$$V = \frac{W}{Q} \tag{86}$$

Electromotive force:

$$\varepsilon = \frac{W}{Q} \tag{87}$$

§14.3 Resistance

Resistance:

$$R = \frac{V}{I} \tag{88}$$

In resistivity,

$$R = \frac{\rho l}{A} \tag{89}$$

Ohm's law:

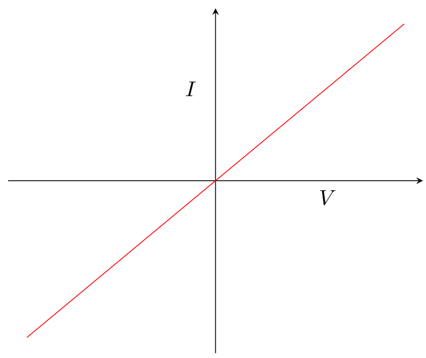
$$I \propto V \tag{90}$$

§14.4 I-V characteristics

An ohmic resistor obeys Ohm's Law. For non-ohmic resistors that do not obey Ohm's Law, factors that cause resistance to deviate are:

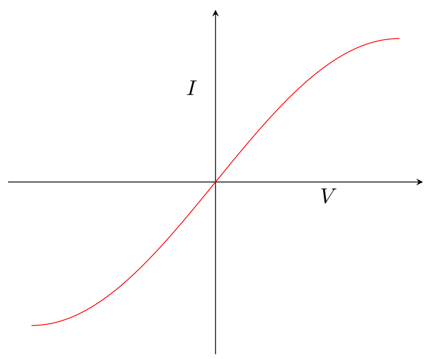
1. Number density of charge carriers n (decrease resistance)
2. Amplitude of atomic vibrations of lattice atoms (increase resistance)

§14.4.1 Ohmic resistor



Resistance is **constant**: as p.d. increases, current increases proportionately.

§14.4.2 Filament lamp

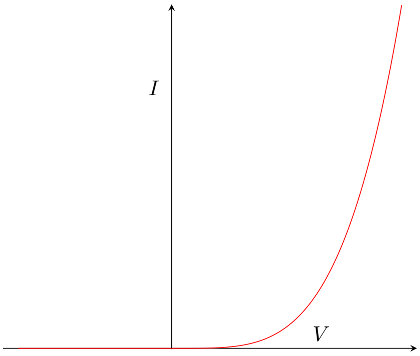


Resistance **increases**: as p.d. increases, current increases less than proportionately.

Useful problem solving technique:

To find a particular resistance value on an $I - V$ graph, draw the $I - V$ graph of an ohmic conductor with the particular resistance value on top of the given graph.

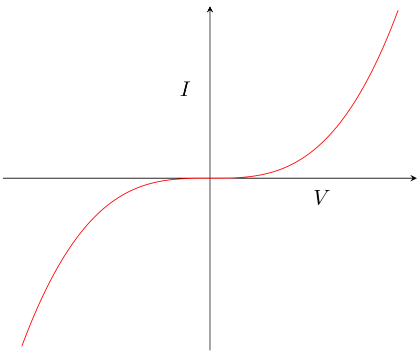
§14.4.3 Semiconductor diode



For forward-biased region, resistance **decreases**: as p.d. increases, current increases more than proportionately.

For reverse-biased region, resistance is **infinitely high**: no current flow through diode.

§14.4.4 Negative Temperature Coefficient (NTC) thermistor



Resistance **decreases**: as p.d. increases, current increases more than proportionately.

Resistance-temperature characteristic: resistance decreases as temperature increases

§14.4.5 Light Dependent Resistor (LDR)

Similar to NTC thermistor.

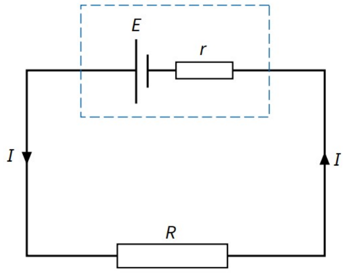
§14.5 Power

Electrical power dissipated by conductor:

$$\begin{aligned} P &= VI \\ P &= I^2R \\ P &= \frac{V^2}{R} \end{aligned} \tag{91}$$

§14.6 Internal resistance

For a real battery, internal resistance $r \neq 0$. Think of internal resistance as another load of resistance r connected with R in series.



Derivations

Transport equation

$$\boxed{I = nAvq}$$

Derivation. Consider a current I passing through a section of a wire of cross-sectional area A.

We define

- n as number density of charge carriers (number per unit volume)
- q as amount of charge of each charge carrier
- v as drift velocity of charge carriers

$$I = \frac{Q}{t} = \frac{Nq}{t} = \frac{nVq}{t} = \frac{nAxq}{t} = nAvq$$

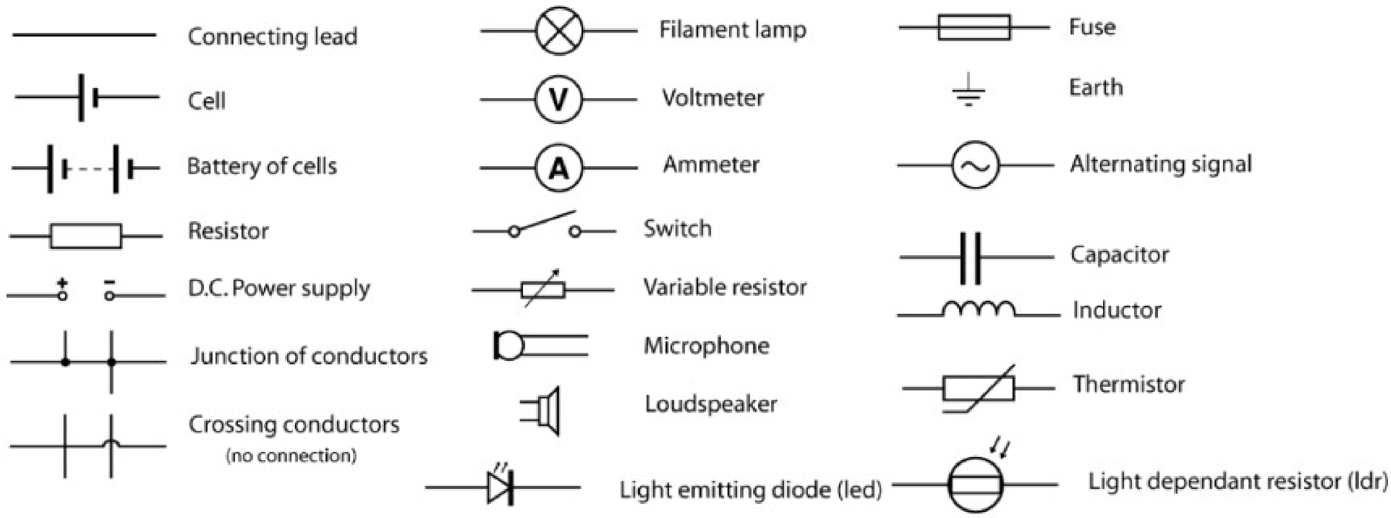
□

§15 D.C. Circuits

Direct current (d.c.): current flowing only in *one direction*

§15.1 Circuit symbols and diagrams

Circuit symbols:



§15.2 Series and parallel arrangements

Current:

$$\text{Series:}\quad I_1 = \cdots = I_n = I$$
$$\text{Parallel:}\quad I = I_1 + \cdots + I_n$$

Voltage:

$$\text{Series:}\quad V_1 + \cdots + V_n = \varepsilon$$
$$\text{Parallel:}\quad \varepsilon = V_1 = \cdots = V_n$$

Resistance:

$$\text{Series:}\quad R_{\text{eff}} = R_1 + \cdots + R_n$$
$$\text{Parallel:}\quad \frac{1}{R_{\text{eff}}} = \frac{1}{R_1} + \cdots + \frac{1}{R_n}$$

Remark. Effective resistance of resistors in parallel is always lower than the lowest resistance in the network.

§15.3 Potential divider

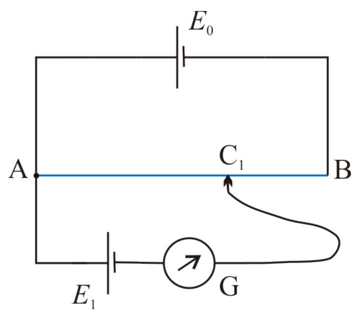
Potential divider rule:

$$V = \frac{R}{R_T} V_T \tag{92}$$

Using potential divider rule along wire,

$$V_{AB} = \frac{\ell_{AB}}{\ell_T} \times V_T.$$

Potentiometer: measures e.m.f. of a source accurately (no current is drawn from the circuit to be measured).



- At balance point, galvanometer registers zero reading, no current flows through galvanometer.
- No p.d. between point A and +ve terminal of E_1, and between point C and −ve terminal of E_1. Hence p.d. between A and C is equal to p.d. across E_1, thus $E_1 = V_{AC}$.
- Since p.d. proportional to length across wire,

$$E_1 = V_{AC} = \frac{\ell_{AC}}{\ell_{AB}} \times E_0$$

§16 Electromagnetism

§16.1 Magnetic Field

Magnetic field lines:

- Field lines represent lines of magnetic force, start from North pole and end at South pole.
- The tangent to magnetic field line at a point gives the direction of the field at that point.
- Density of field lines indicates field strength.

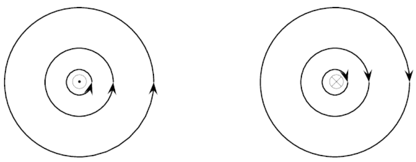
Right hand grip rule: determine direction of magnetic field due to current

Magnetic flux patterns

- Due to current in long straight wire:

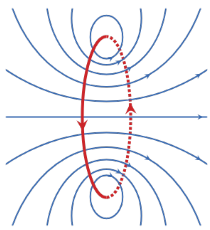
$$B = \frac{\mu_0 I}{2\pi r} \tag{93}$$

where $\mu_0 = 4\pi \times 10^7\ \mathrm{H\,m^{-1}}$ is vacuum magnetic permeability constant.



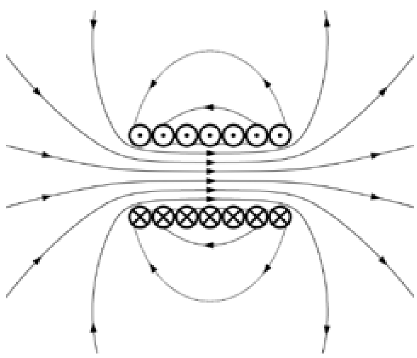
- Due to current in flat circular coil:

$$B = \frac{\mu_0 N I}{2r} \tag{94}$$



- Due to current in long solenoid:

$$B = \mu_0 n I \tag{95}$$



§16.2 Magnetic Force

§16.2.1 Force on current-carrying conductor

Magnitude of force:

$$F = BIL\sin\theta \tag{96}$$

where θ is angle between magnetic field and current.

Fleming's left hand rule: determine direction of the force

§16.2.2 Force between current-carrying conductors

Currents in same direction: attract each other

Currents in opposite direction: repel each other

Force by conductor with current I_1 on conductor with current I_2 is

$$F_1 = BI_1L = \frac{\mu_0 I_2}{2\pi r} I_1 L \tag{97}$$

§16.2.3 Force on a moving charge

Consider charge q travelling at constant speed v at an angle θ to magnetic field of flux density B. Assume charge travels a distance L in time t, so $v = \frac{L}{t}$ thus $L = vt$.

Substituting $I = \frac{q}{t}$ and $L = vt$ into eq. (96) gives

$$F = Bqv\sin\theta. \tag{98}$$

§16.3 Applications

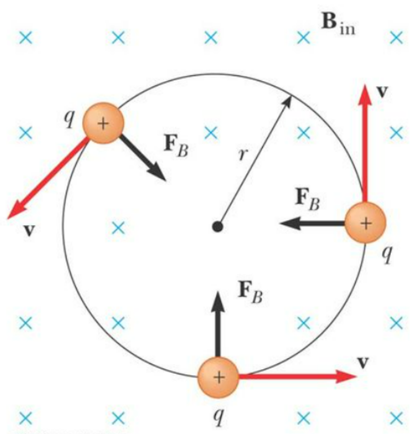
§16.3.1 Circulating charge

If v and B are *perpendicular*, charge undergos <u>uniform circular motion</u>.

- By Fleming's left hand rule, magnetic force acting on charge is always perpendicular to its velocity.
- Causes change in direction without change in speed.
- Magnetic force acting on charge provides centripetal force for charge to undergo uniform circular motion.

$$F_B = F_c \implies Bqv = \frac{mv^2}{r} \implies \boxed{r = \frac{mv}{Bq}}$$

Charge of higher speed has larger radius of curvature.

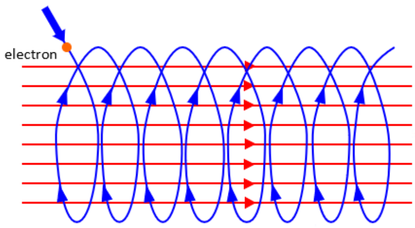


Period of revolution is independent of v:

$$T = \frac{2\pi r}{v} = \frac{2\pi\left(\frac{mv}{Bq}\right)}{v} = \frac{2\pi m}{Bq}.$$

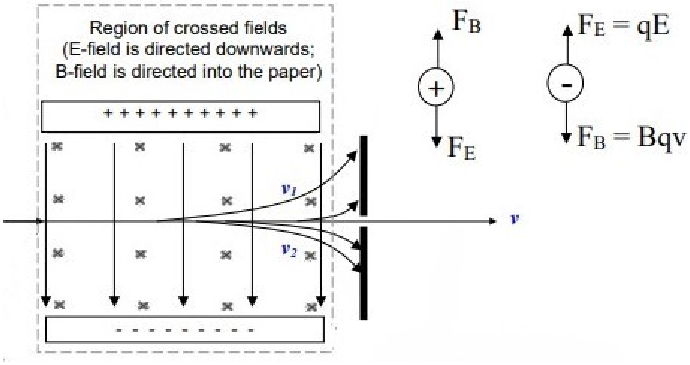
If v and B are not perpendicular, charge moves in **helical path**:

- Component of velocity perpendicular to magnetic field results in circular motion in the plane perpendicular to magnetic field
- Component of velocity parallel to magnetic field causes charge to move at constant speed in same/opposite direction to magnetic field, perpendicular to plane of circular motion



§16.3.2 Crossed fields

Crosssed fields consist of uniform E and B fields set up perpendicular to each other.



Beam of charged particles (electrons) with a range of velocities pass through crossed field.

- By Fleming's LHR, electron experiences upward electric force and downward magnetic force[10].
- To pass through undeflected, electric force and magnetic force are equal in magnitude, opposite in direction:

$$F_B = F_E \implies Bqv = qE \implies \boxed{v = \frac{E}{B}}$$

Hence cross fields are used in velocity selectors, as only charges with a specific velocity can pass through undeflected.

[10] the reverse for positively charged particles

§17 Electromagnetic Induction

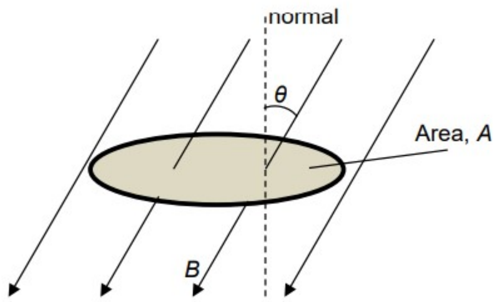
Electromagnetic induction: e.m.f. induced due to changing magnetic field.

§17.1 Magnetic flux

Magnetic flux:

$$\Phi = BA\cos\theta \tag{99}$$

where θ is angle between normal of plane and magnetic field.



Remark. When B not perpendicular to A, resolve B to find component of B parallel to normal of plane.

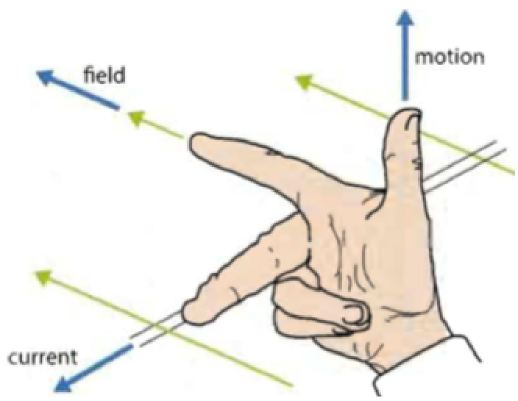
Magnetic flux linkage:

$$N\Phi = NBA\cos\theta \tag{100}$$

§17.2 Laws of electromagnetic induction

$$\varepsilon = -\frac{\mathrm{d}(N\Phi)}{\mathrm{d}t} \tag{101}$$

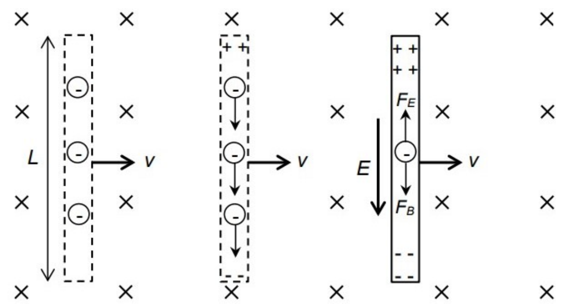
Fleming's right hand rule: determine direction of *induced* e.m.f.



Answering format for qualitative questions:

1. Increase (decrease) in magnetic flux due to (reason).
2. By Faraday's law, e.m.f. induced in the ...
3. *By Lenz's law, direction of induced current will flow ... to produce magnetic field into/out of the paper to oppose the decrease (increase) in magnetic flux.

§17.2.1 Motional e.m.f.



- Conductor moves right, electrons in conductor move right, so current flows left. By Fleming's LHR, electrons experience downward magnetic force.
- Due to this force, electrons move towards lower end of conductor. As a result of charge separation, upper end has higher potential than lower end, thus generates induced e.m.f.
- Electric field (directed downward) produced inside conductor. Charges accumulate at both ends until downward magnetic force is balanced by upward electric force on electrons.

  $$F_E = F_B \implies qE = Bqv \implies E = vB$$

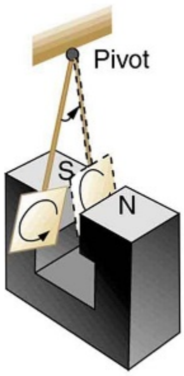
  Since electric field in conductor is uniform, p.d. across ends of conductor of length L is $\Delta V = EL$, so

  $$\varepsilon = BLv \tag{102}$$

Remark. Induced e.m.f. is proportional to rate of flux cutting, as $\varepsilon \propto v$.

§17.3 Applications

§17.3.1 Eddy currents

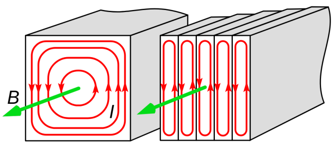


- Solid plate conductor enters magnetic field, flux through plate increases. **Eddy current** induced, flows anticlockwise to provide its own magnetic field out of the page to decrease flux.

  The field in the plate is not uniform and the rate of cutting is not the same over the whole plate, so different e.m.f. are induced in different parts of plate. Hence eddy currents flow simultaneously along many different paths in swirls.

- **Drawbacks**: By conservation of energy, induced currents do work and raise temperature of iron core, heat dissipated as eddy currents flow in conductor. Thus loss of energy in applications.

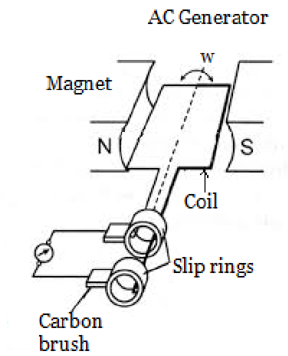
  Cut slits in the plate (laminate) to eliminate paths for current flow, prevent large eddy currents from forming.



- **Applications**: set up braking system which can rapidly convert KE to other forms of energy e.g. stop roller coasters, galvanometers, voltmeters, ammeters.

§17.3.2 Generator

Electric generators convert mechanical energy into electrical energy.



When a coil of N turns rotates with constant angular velocity ω in uniform magnetic field of flux density B, an e.m.f. ε is induced as θ changes.

$$\varepsilon = -\frac{\mathrm{d}(N\Phi)}{\mathrm{d}t} = -\frac{\mathrm{d}(NBA\cos\theta)}{\mathrm{d}t} = NBA\omega\sin\theta$$

where $\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}$.

Hence flux-time graph is a cosine graph, while e.m.f-time graph is a sine graph; both graphs are out of phase.

Since direction of e.m.f. changes with time, alternating current is generated.

§17.3.3 Transformer

Refer to next topic.

§18 Alternating Current

§18.1 Terminology

Alternating current (a.c.): current that varies periodically with time in magnitude and direction.

$$\omega = 2\pi f = \frac{2\pi}{T}$$

Graphically,

$$I_{\text{r.m.s.}} = \sqrt{\langle I^2 \rangle} = \sqrt{\frac{\int_0^T I^2 \, \mathrm{d}t}{T}} \tag{103}$$

where $\int_0^T I^2 \, \mathrm{d}t$ is the area under $I^2 - t$ graph over one cycle.

a.c. is usually in **sinusoidal** form:

$$\begin{aligned} I &= I_0 \sin \omega t \\ V &= V_0 \sin \omega t \end{aligned} \tag{104}$$

§18.1.1 Mean power

For <u>sinusoidal</u> alternating current, mean power is

$$\begin{aligned} \langle P \rangle &= \langle I^2 \rangle R \\ &= \langle {I_0}^2 \sin^2 \omega t \rangle R \\ &= {I_0}^2 \langle \sin^2 \omega t \rangle R \\ &= {I_0}^2 \left(\frac{1}{2} \right) R = \frac{1}{2} {I_0}^2 R = \frac{1}{2} P_0. \end{aligned}$$

Hence mean power is <u>half the maximum power</u>:

$$\langle P \rangle = \frac{1}{2} P_0 \tag{105}$$

Remark. Mean power is useful in calculating energy dissipated in a given period of time, since instantaneous power changes with time.

§18.1.2 Root-mean-square value

Rewriting $P = I^2 R$,

$$\begin{aligned} \langle I^2 R \rangle &= \frac{1}{2} {I_0}^2 R \\ \langle I^2 \rangle &= \frac{1}{2} {I_0}^2 \end{aligned}$$

Taking square root on both sides,

$$I_{\text{r.m.s.}} = \frac{1}{\sqrt{2}} I_0 \tag{106}$$

Similarly,

$$V_{\text{r.m.s.}} = \frac{1}{\sqrt{2}} V_0 \tag{107}$$

To find the current by d.c. source that dissipates the same power in a resistor as the mean power dissipated by a.c., we equate

$$\begin{aligned} P_{\text{d.c.}} &= \langle P \rangle \\ {I_{\text{d.c.}}}^2 R &= \langle I^2 R \rangle = \langle I^2 \rangle R \\ {I_{\text{d.c.}}}^2 &= \langle I^2 \rangle \\ I_{\text{d.c.}} &= \sqrt{\langle I^2 \rangle} = I_{\text{r.m.s.}} \end{aligned}$$

Hence a d.c. of magnitude $I_{\text{r.m.s.}}$ dissipates the same power as mean power of a.c.

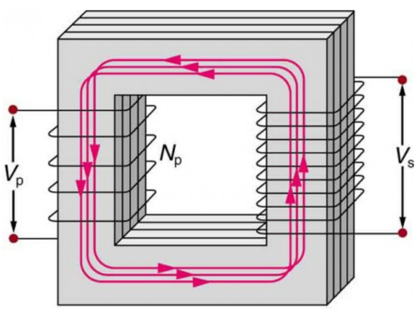
Also,

$$\langle P \rangle = \frac{1}{2} P_0 = \frac{1}{2} V_0 I_0 = \frac{V_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}}$$

so

$$\begin{aligned} \langle P \rangle &= V_{\text{r.m.s.}} I_{\text{r.m.s.}} \\ &= {I_{\text{r.m.s.}}}^2 R \\ &= \frac{{V_{\text{r.m.s.}}}^2}{R} \end{aligned} \tag{108}$$

§18.2 Transformer



Principle of operation

1. a.c. source causes alternating current in primary coil, which sets up alternating magnetic flux in iron core.
2. Alternating magnetic flux is linked from primary to secondary coil.
3. By Faraday's law, alternating e.m.f. induced in secondary coil.
4. Alternating e.m.f. in secondary coil gives rise to a.c. in secondary coil.

Remark. All currents and e.m.f.s have same frequency as a.c. source.

Turns ratio:

$$\frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{I_p}{I_s} \tag{109}$$

- Step-up transformer: $N_p < N_s$
- Step-down transformer: $N_p > N_s$

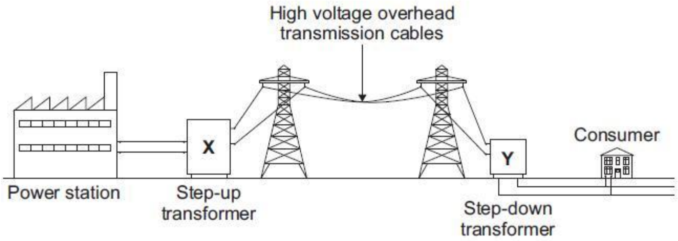
Remark. Power is conserved.

Power loss:

Source	Minimise
Joule heating: wires used for windings of coils have resistance and so heating occurs, resulting in power loss $P = I^2R$.	Use thicker wires made of material with low resistivity e.g. copper
Eddy currents: alternating magnetic flux induces eddy currents in iron core, causes heating.	Laminate iron core, which reduces area of circuits in the core, and thus reduces e.m.f. induced and current flowing within the core
Hysteresis loss: magnetisation of core is repeatedly reversed by alternating magnetic field, energy required to magnetise the core (while the current is increasing) is not entirely recovered during demagnetisation, thus difference in energy is lost as heat in the core.	Use magnetic material with low hysteresis loss e.g. soft iron
Flux leakage: flux due to primary may not all link to the secondary coil if it is badly designed or has air gaps in it, so flux "leaked" to the surrounding, power is lost, thus not all the power from primary coil can be transferred to secondary coil	Use core of high magnetic permeability e.g. soft iron to strongly concentrate magnetic field produced by primary coil, result in in large changes in magnetic flux linkage in secondary coil

§18.2.1 Transmission of electricity

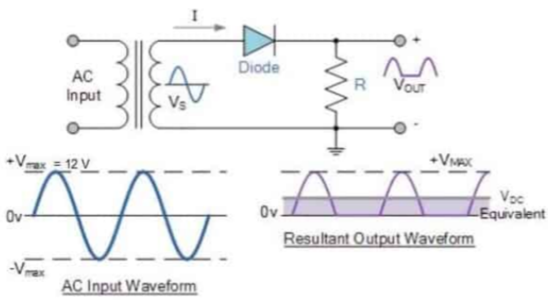
a.c. used to transmit power: minimise power loss by stepping up voltage using transformer, transmit power at high voltage, then step down voltage for household use



§18.3 Rectification with diode

Rectification is conversion of a.c. to d.c.

- First half cycle: diode is forward-biased, current flows in circuit, p.d. across load almost has same value as input p.d.
- Second half cycle: diode is reverse-biased, no current flows in circuit, p.d. across load is zero.



Part VI

Modern Physics

§19 Quantum Physics

§19.1 Energy of a photon

Energy of one photon is

$$E = hf \tag{110}$$

where $h = 6.63 \times 10^{-34}$ J s is Planck's constant.

By $c = f\lambda$,

$$E = \frac{hc}{\lambda}.$$

1 electronvolt: energy gained by an electron when it is accelerated through potential difference of 1 V.

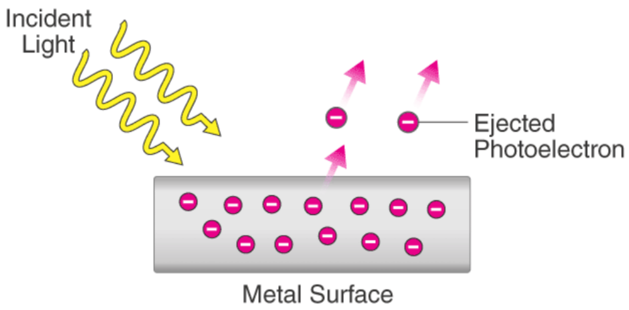
$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

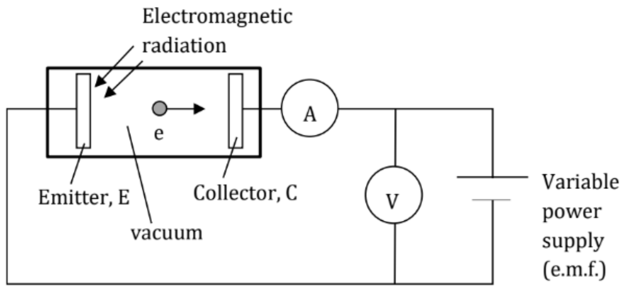
Intensity of EM radiation is

$$I = \frac{P}{\text{Area}} = \frac{Nhf}{t \times \text{Area}}$$

where N is number of photons.

§19.2 Photoelectric effect





Experimental observations from the photoelectric effect experiment (**evidence for particulate nature of light**):

Observation	Wave model	Photon model
Instantaneous emission of electrons (even at low intensity)	high intensity of radiation should be needed to have immediate effect	a single photon is enough to release one electron
No emission of electrons below threshold frequency	any frequency can give rise to emission if exposure time is long enough	a low frequency photon has energy less than work function, so no release of electron
Max KE of electrons independent of intensity	greater intensity means more energy so electrons should have more energy	greater intensity does not mean more energetic photons, so electrons cannot have more energy
Max KE dependent on frequency	should be increasing intensity and not frequency that increases energy of electrons	higher frequency means more energetic photons, by $E = hf$, so electrons gain more energy
Rate of emission of electrons dependent on intensity	greater intensity so more energy, more electrons emitted	greater intensity means more photons per second, so more electrons emitted per second

Explanation of photoelectric effect:

1. A photon of energy hf is incident on metal surface.
2. Photon meets an electron (one-to-one electron). Electron absorbs all the energy of photon.
   Note that not all photons (of sufficient energy) get to interact with electrons.
3. Some energy used to overcome forces of attraction to escape metal, rest is KE of emitted electrons.
   Photoelectrons are emitted in all random directions with varying speeds.

Einstein's photoelectric equation:

$$hf = \Phi + \mathrm{KE}_{\max} \tag{111}$$

Work function Φ of a metal: minimum energy of photon to cause emission of electron from metal surface[11]

Threshold frequency f_0: lowest frequency of EM radiation that gives rise to emission of electron from metal surface

Threshold wavelength λ_0: highest wavelength of EM radiation that gives rise to emission of electron from metal surface

At threshold frequency, KE = 0 by definition. Thus

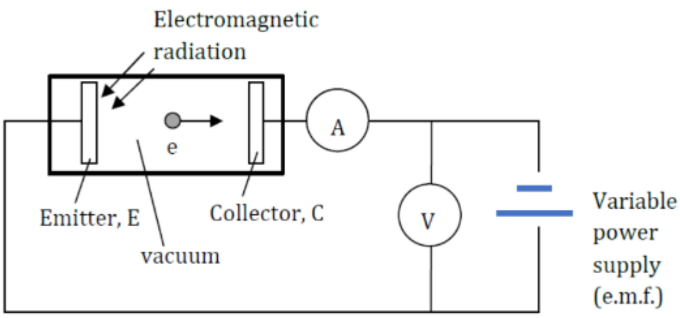
$$hf_0 = \frac{hc}{\lambda_0} = \Phi.$$

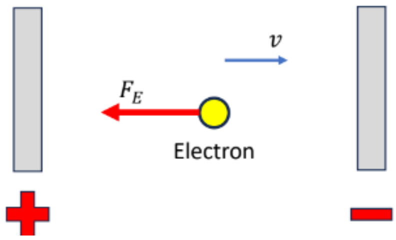
When frequency is less than f_0, electron cannot escape from metal, energy gained converted to KE, collide with metal ions and lose KE, which warms up metal.

Remark. Emitted electrons are likely to have a range of KE:

- Maximum KE corresponds to electrons emitted from surface.
- Photon may interact with electrons not at surface. These electrons require energy to be brought to the surface.

Determine max KE of emitted electrons:





- Reverse polarity of battery - electrons repelled away from the collector
- If potential difference is high enough, electrons will be stopped from reaching the collector (zero photocurrent)
- **Stopping potential** V_s: minimum potential applied to stop the most energetic electrons

From conservation of energy,

$$\begin{aligned} \text{Work done} &= \Delta\mathrm{KE} \\ qV &= \mathrm{KE}_f - \mathrm{KE}_i \\ e(-V_s) &= 0 - \mathrm{KE}_{\max} \end{aligned}$$

$$\mathrm{KE}_{\max} = eV_s \tag{112}$$

Substituting eq. (112) into eq. (111) gives

$$hf = \Phi + eV_s. \tag{113}$$

[11] depends on nature of material of metal, surface condition

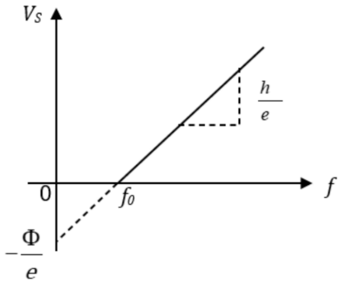


Figure 1: V_s against f graph

$$V_s = \frac{h}{e} f - \frac{\Phi}{e}$$

Given $\Phi_2 > \Phi_1$,

- horizontal intercept is larger because $(f_0)_2 > (f_0)_1$ since $\Phi = hf_0$,
- vertical intercept is lower since $-\frac{\Phi_2}{e} < -\frac{\Phi_1}{e}$,
- gradient is the same since $\frac{h}{e}$ is constant.

Stopping potential is independent of intensity

- As intensity increases, number of photoelectrons emitted increases.
- However, maximum velocity attained by them remains independent of intensity of radiation, only depends on frequency of radiation.

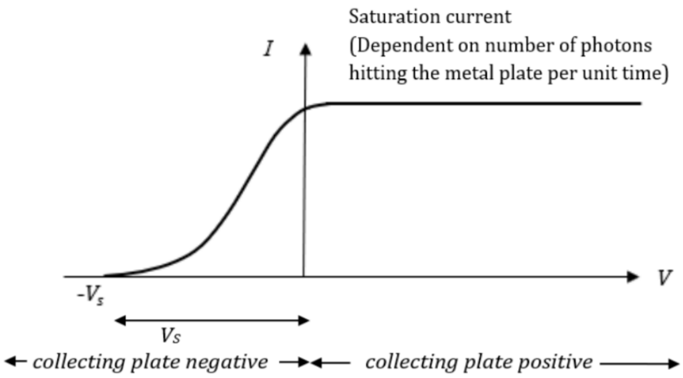


Figure 2: $I - V$ graph

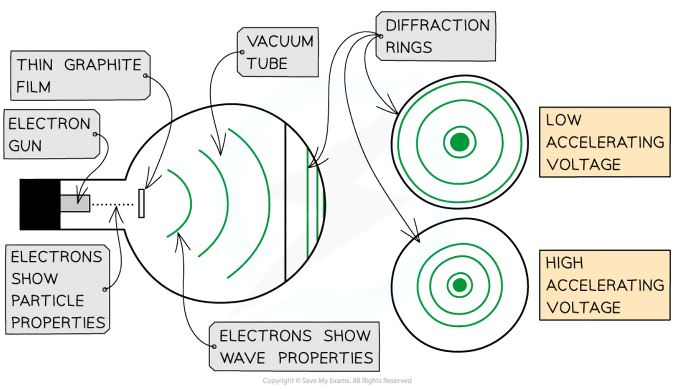
- Stopping potential: negative collector voltage repels all photoelectrons so zero photocurrent (depends on frequency of incident radiation, by eq. (113))
- Negative bias V: rate of photoelectrons reaching collector increases, so photocurrent increases
- Positive bias V: limited number of photons incident per unit time for a particular intensity of EM radiation, which limits number of photons that can undergo a 1:1 interaction to release photoelectron per unit time - all emitted photoelectrons are collected, reaches saturation current
- Photoelectric current proportional to intensity: at higher intensity, higher rate at which photons are incident on metal surface, thus higher rate of photoelectron emission and current

§19.3 Wave–particle duality

Wave–particle duality of EM radiation: can behave as (1) waves in some situations, (2) particles in others

Electron diffraction experiment provides evidence for wave nature of particles.

- Lattice of carbon atoms acts as diffraction grating
- Electrons posssess wave propeties with de Broglie wavelength $\lambda = \frac{h}{p}$, so undergo diffraction through graphite film, gives interference pattern (bright and dark rings).



de Broglie wavelength:

$$\lambda = \frac{h}{p} \tag{114}$$

By KE $= \frac{p^2}{2m}$,

$$\lambda = \frac{h}{\sqrt{2m(\text{KE})}}$$

Photons incident on a surface experience change in momentum. By N2L and N3L, they exert a force on the surface, resulting in a pressure on the surface, known as **radiation pressure**.

§19.4 Energy levels in atoms

Electrons orbiting nucleus exist at discrete energy levels.

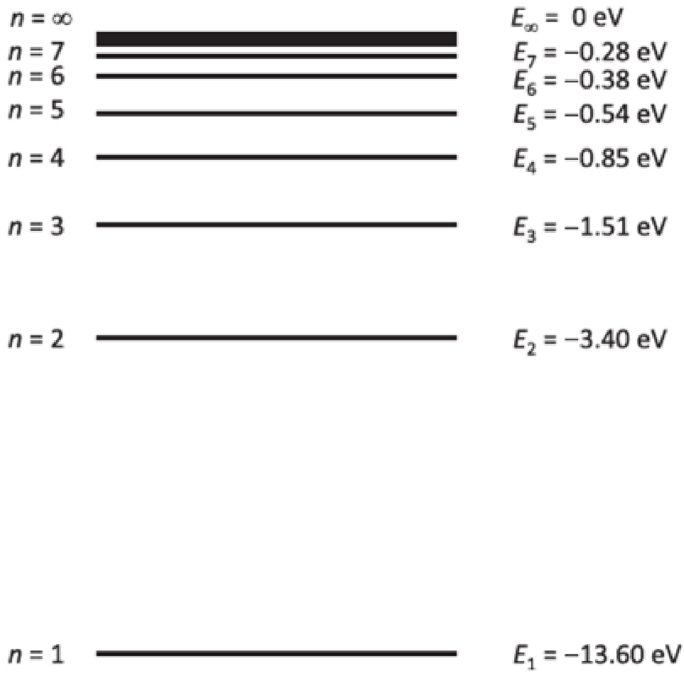


Figure 3: Energy level diagram

- At highest energy level $n = \infty$, electron no longer bound to atom, assigned $E_\infty = 0$ eV

  Ionisation energy: minimum energy required to remove electron completely from atom (transition from ground state to infinite level)

- Lower energy level, more negative energy value – more stable states
- Energy difference between any two adjacent levels gets smaller as n increases, higher energy levels converge to a continuum

Excitation: (absorb energy)

1. Photon absorption: electron absorbs all energy from one photon, energy of photon must be EXACTLY equal to difference between energy levels (else photon not absorbed):
$$hf = |\Delta E|$$
2. High speed collision: electron absorbs a portion or all KE of colliding particle, kinetic energy of colliding particle must be enough (rest converted to KE):
$$\text{KE}_{\text{colliding}} \geq |\Delta E|$$

De-excitation: (emit energy)

Energy of photon released EXACTLY equals to difference between energy levels:

$$hf = |\Delta E|$$

One photon emitted for each transition made.

Remark. For transition between n energy levels to ground state, number of unique photons emitted is $\binom{n}{2}$.

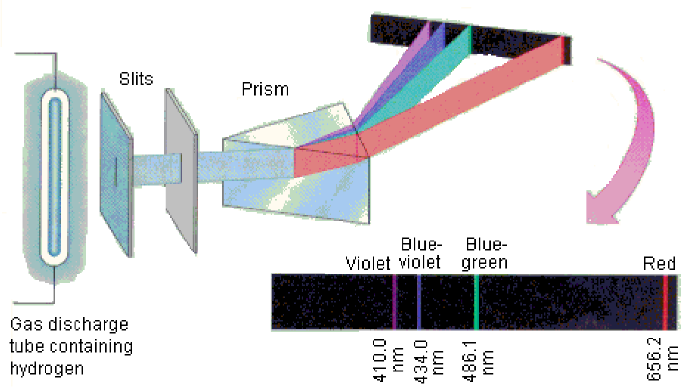
§19.5 Line spectra

Emission/absorption of light of specific wavelength from a continuous spectra is evidence for discrete energy levels in atoms as photons of specific wavelength when electrons are excited/de-excited.

Remark. Application: identify elements (line spectra of each element is unique, as each element has unique profile of discrete energy levels)

§19.5.1 Emission line spectra

Emission line spectra: discrete bright lines of different colours on a dark background

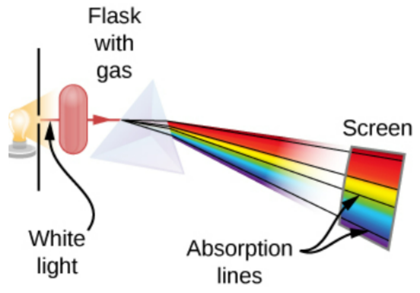


1. Low pressure gas is excited by high voltage. Excited state is unstable, so electron will be de-excited.
2. When electron transits from higher to lower energy states, photons with specific frequency/wavelength will be emitted such that $hf = |\Delta E|$.
3. When passing through diffraction grating, photons are diffracted at specific angles according their wavelengths, from $d \sin\theta = n\lambda$.

   This gives rise to set of characteristic spectral lines.

§19.5.2 Absorption line spectra

Absorption line spectra: continuous spectrum crossed by dark lines



1. White light with continuous range of wavelengths is passed through a cool gas to excite gas from the ground state.

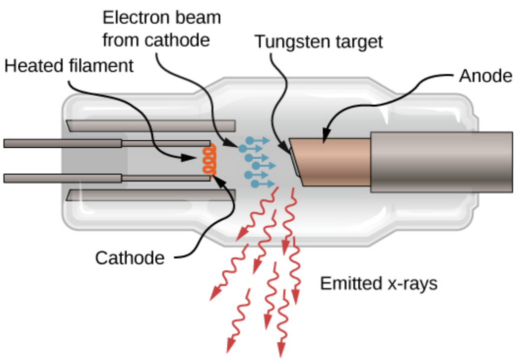
2. For electrons to be excited to excited state, photons with specific frequency/wavelength will be absorbed such that $hf = |\Delta E|$.

3. When electrons are de-excited, they are emitted in random directions, so "missing" from the straight through direction.

   This is observed as black lines on the spectrum.

§19.6 X-ray

Braking radiation: EM radiation produced by deceleration of charged particle (KE lost converted to photons emitted)



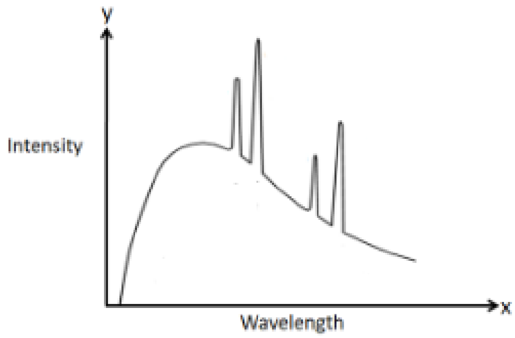
X-ray production:

1. Filament heated up, emits electrons
2. Electrons accelerated by p.d., gain KE:
$$\text{KE} = eV$$
3. Electron collide with metal target, decelerates
4. Energy lost by electrons = energy of X-ray photon emitted

Remark. X-ray tube is *vacuum*, to prevent electrons emitted from colliding with air molecules, lose KE.

Remark. Electrons emitted have a range of speeds, because electrons are emitted in different directions, thus different initial velocities & electric force acting on electrons are horizontal and constant in magnitude, only causes increase in horizontal component of velocity of electrons.

X-ray spectrum:



1. **Broad continuous spectrum**
   - When electrons hit metal target, decelerated by different extents, so differing amounts of KE
   - If electron gives up *all* its energy in collision, max energy X-ray photon has wavelength λ_{min}:
   $$\frac{hc}{\lambda_{\text{min}}} = \text{KE}_{\text{max}} = eV$$
2. **Characteristic lines**
   - Charged particle collide with inner shell electron, ejects it, create vacancy in lower shell, electrons from higher energy shells de-excite
   - X-ray photon produced have specific wavelength: energy of characteristic X-ray photon = energy difference between energy levels
   $$\frac{hc}{\lambda} = \Delta E$$

§19.7 Uncertainty principle

Heisenberg uncertainty principle: impossible to simultaneously measure exact position and exact momentum

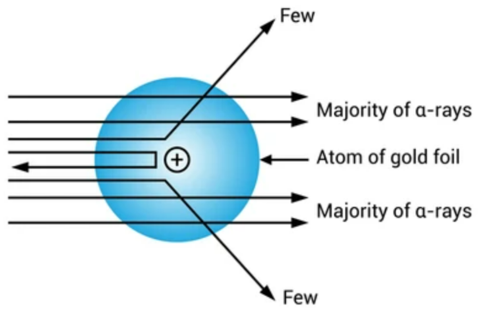
$$\Delta p \Delta x \geq h \tag{115}$$

§20 Nuclear Physics

Particle	Charge	Mass
proton	$+e$	m_p
neutron	0	$\approx m_p$
α**-particle** ($^4_2$He nucleus)	$+2e$	$\approx 4m_p$
β**-particle** ($^{\ 0}_{-1}$e electron)	$-e$	$\frac{1}{1800}m_p$
antineutrino $\bar{\nu}$	0	0
γ**-particle** (photon)	0	0

§20.1 The nucleus

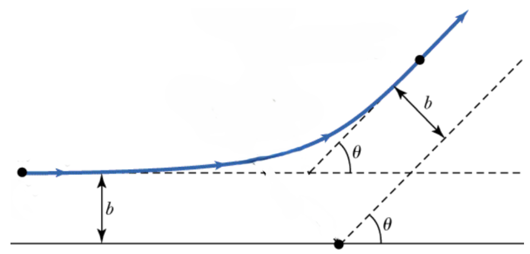
Rutherford's alpha particle scattering experiment:



1. Most α-particles travel through undeflected
   $\hookrightarrow$ Atom is mostly empty space (size of nucleus is small compared to the atom)
2. Small but significant percentage of α-particles are deflected by large angles
   $\hookrightarrow$ Nucleus is charged
3. Occasionally, an α-particle is deflected 180°
   $\hookrightarrow$ Mass is concentrated in the nucleus

Drawing of deflection pathway:

- Hyperbolic path
- Deflection is early and gradual, not a sudden change in direction.
- Impact parameter is constant
- Asymptotes



§20.2 Mass defect and nuclear binding energy

Einstein's **mass–energy equivalence**:

$$E = mc^2 \tag{116}$$

Mass defect Δm: difference between mass of unbounded nucleons & nucleus

$$\Delta m = \underbrace{Zm_p + (A - Z)m_n}_{\text{unbounded nucleons}} - \underbrace{m_{\text{nucleus}}}_{\text{nucleus}} \tag{117}$$

Remark. Mass of a single nucleus is ALWAYS less than total mass of its unbounded nucleons, because energy is supplied to overcome strong nuclear force holding nucleons together.

Remark. We can think of $^2_1\text{H} + {}^2_1\text{H} \longrightarrow {}^4_2\text{He}$ as a two step process:

$$^2_1\text{H} + {}^2_1\text{H} \xrightarrow{E_{\text{supplied}}} 2p + 2n \xrightarrow{E_{\text{released}}} {}^4_2\text{He}$$

Compare m_{products} and $m_{\text{reactants}}$: if $m_{\text{products}} > m_{\text{reactants}}$, energy supplied converted to extra mass, thus reaction is endothermic, and vice versa.

$$\begin{aligned} E &= E_{\text{released}} - E_{\text{supplied}} \\ &= \sum \text{BE}_{\text{products}} - \sum \text{BE}_{\text{reactants}} \end{aligned}$$

Endothermic reaction: energy supplied, so $m_{\text{products}} > m_{\text{reactants}}$ and $\sum \text{BE}_{\text{products}} < \sum \text{BE}_{\text{reactants}}$

Exothermic reaction: energy released, so $m_{\text{products}} < m_{\text{reactants}}$ and $\sum \text{BE}_{\text{products}} > \sum \text{BE}_{\text{reactants}}$

§20.3 Nuclear processes

Nuclear processes obey 4 important conservation laws

1. Conservation of nucleon number
2. Conservation of proton number
3. Conservation of linear momentum
4. Conservation of mass-energy

§20.3.1 Nuclear reactions (non-spontaneous)

Nuclear reaction: non-spontaneous, caused by external stimulus e.g. collision of two or more nuclides

Nuclear equation:

$$^{14}_{7}\text{N} + {}^{4}_{2}\text{He} \longrightarrow {}^{17}_{8}\text{O} + {}^{1}_{1}\text{H}$$

Remark. Balance number of protons and neutrons.

Binding energy per nucleon: Average energy needed to remove a nucleon from the nucleus: BE of nucleus divided by no. of nucleons.

Nuclear stability:

- Higher BE/nucleon: nucleons bounded more closely, more energy on average needed to separate nucleons from nucleus →nucleus more stable

- Lower BE/nucleon: nucleons bounded less closely, less energy on average needed to separate nucleons from nucleus →nucleus less stable

Remark. Reactions favour formation of stable products.

Remark. Since more stable products are formed, $\sum \text{BE}_{\text{products}} > \sum \text{BE}_{\text{reactants}}$, thus energy is released, so fission and fusion are exothermic.

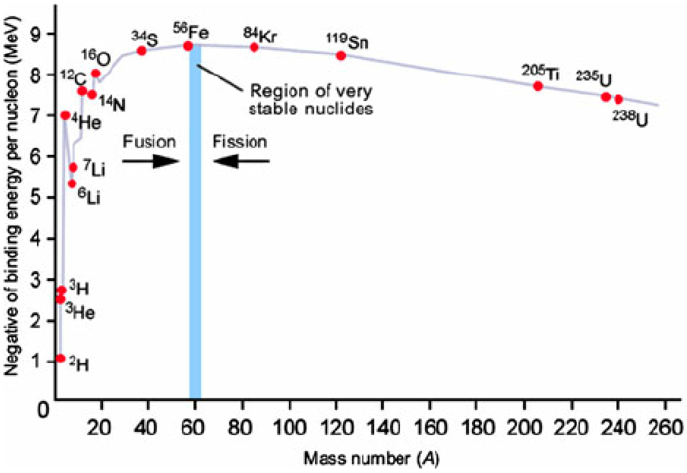


Figure 4: Binding energy per nucleon against nucleon number

Calculating energy released:

(i) $E_{\text{released}} = (m_{\text{reactants}} - m_{\text{products}}) \times c^2$

(ii) $E_{\text{released}} = \sum \text{BE}_{\text{products}} - \sum \text{BE}_{\text{reactants}}$

(iii) $E_{\text{absorbed}} = (m_{\text{products}} - m_{\text{reactants}}) \times c^2$

(iv) $E_{\text{absorbed}} = \sum \text{BE}_{\text{reactants}} - \sum \text{BE}_{\text{products}}$

Note that the mass difference in (i) and (iii) should not be called "mass defect", by definition.

§20.3.2 Radioactive decay (spontaneous)

Alpha decay	Beta decay	Gamma decay
α-particle is helium nucleus $^{4}_{2}\text{He}$	β-particle is electron $^{0}_{-1}\text{e}$	photon
$^{A}_{Z}\text{X} \longrightarrow {}^{A-4}_{Z-2}\text{Y} + {}^{4}_{2}\text{He}$	$^{A}_{Z}\text{X} \longrightarrow {}_{Z+1}^{A}\text{Y} + {}^{0}_{-1}\text{e} + \bar{\nu}$ An unstable neutron in nucleus turns into proton and electron, emitting the electron.	$^{A}_{Z}\text{X}^* \longrightarrow {}^{A}_{Z}\text{X} + \gamma$ Emitted when nucleus changes from excited (nuclear) state to lower energy (nuclear) state.
strong ionising strength	weak ionising strength	very weak ionising strength
penetrating ability: stopped by a piece of paper	penetrating ability: stopped by few mm of aluminium	penetrating ability: stopped by 10 cm of lead

Antineutrino in beta decay

- If two decay products, by conservation of momentum,
$$p_Y + p_e = 0 \implies |p_Y| = |p_e|.$$
By conservation of energy, energy released converted to KE of products:
$$\begin{aligned} E_{\text{released}} &= \text{KE}_Y + \text{KE}_e \\ &= \frac{p_Y^2}{2m_Y} + \frac{p_e^2}{2m_e} \\ &= \frac{p_e^2}{2m_e}\frac{m_e}{m_Y} + \frac{p_e^2}{2m_e} \\ &= \text{KE}_e\left(1 + \frac{m_e}{m_Y}\right) \end{aligned}$$
thus
$$\text{KE}_e = \left(\frac{m_Y}{m_Y + m_e}\right)E$$
which is fixed; hence β-particle is mono-energetic.
Remark. Since $m_Y \gg m_e$, $\frac{m_Y}{m_Y+m_e} \approx 1$, so β-particle contains most of the energy.

- However, it was determined experimentally that energy of β-particles have a continuous rather than a discrete spectrum.
It was proposed that another particle (**antineutrino**) emitted, so that energy is shared between β-particle and antineutrino, such that β-particles have a range of energies and momenta, by conservation of mass-energy and linear momentum.
Antineutrino is neutrally charged (by conservation of charge).

Activity:
$$A \coloneqq -\frac{\mathrm{d}N}{\mathrm{d}t} \tag{118}$$

Decay constant:
$$\lambda \coloneqq \frac{-\frac{\Delta N}{N}}{\Delta t} \tag{119}$$
where $\frac{\Delta N}{N}$ is the fraction of radioactive nuclei in a sample that has decayed in the small time interval Δt.

Combining eq. (118) and eq. (119) gives
$$A = \lambda N. \tag{120}$$

Combining eq. (118) and eq. (120) gives
$$\lambda N = -\frac{\mathrm{d}N}{\mathrm{d}t}$$
on solving which gives
$$N = N_0 e^{-\lambda t} \tag{121}$$
and thus
$$A = A_0 e^{-\lambda t}. \tag{122}$$

From definition,
$$t_{1/2} = \frac{\ln 2}{\lambda}. \tag{123}$$

A useful formula is
$$N = N_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}. \tag{124}$$

Count rate: no. of ionising particles detected per unit time

Geiger-Muller counter: used to detect ionising particles

Background radiation: ionising radiation that is always present in the environment (systematic error)

- Artificial sources: medical X-rays, cigarettes
- Natural sources: cosmic radiation, carbon-14 and potassium-40 in human bodies from birth

Remark. Deduct background count C_b from C to get true count rate $C - C_b$.

§20.4 Biological effects of radiation

Ionising radiation with sufficient energy can remove electron from atom, causing it to be charged (ionised).

- Direct effects of ionising radiation on cells
Radiation damanges DNA (deoxyribo-nucleic acid) strands through breaks and mutations.
- Indirect effects of ionising radiation on cells
Radiation interacts with other molecules, e.g. water, producing ions and radicals (H^+, OH^-, $H\cdot$, $OH\cdot$) which can then attack cells and DNA. They can also combine to form toxic substances e.g. H_2O_2.
- Consequence of cell damage
  1. Cell dies: (which is if not too many cells die)
  2. Cell repairs itself (which is good)
  3. Cell survives but mutates (which is bad because it may cause cancer)
  Acute effects (high doses of radiation over short time): Symptoms include vomiting, burns, blood count change, hair loss, sterility and death.
  Chronic (low doses of radiation over long time): Development of cancer, genetic mutation, developmental abnormalities and growth disorders.