Chapter 3 – Forces

Subject content

Content • Balanced and unbalanced forces • Free-body diagram • Friction **Learning outcomes** Candidates should be able to: (a) apply Newton's Laws to: (i) describe the effect of balanced and unbalanced forces on a body (ii) describe the ways in which a force may change the motion of a body (iii) identify action-reaction pairs acting on two interacting bodies (stating of Newton's Laws is not required) (b) identify forces acting on an object and draw free-body diagram(s) representing the forces acting on the object (for cases involving forces acting in at most 2 dimensions) (c) solve problems for a static point mass under the action of 3 forces for 2-dimensional cases (a graphical method would suffice) (d) recall and apply the relationship resultant force = mass × acceleration to new situations or to solve related problems (e) explain the effects of friction on the motion of a body

Definitions

Phrase	Definition
Force	Push / pull that one object exerts on another object Interaction between two objects or between an object and its environment
1 newton (1 N)	Force that produces acceleration of 1 ms^{-2} on mass of 1 kg
Friction	Contact force that opposes / tends to oppose motion between surfaces in contact

Newton's laws of motion

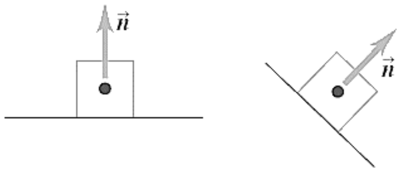
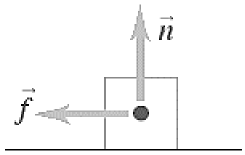
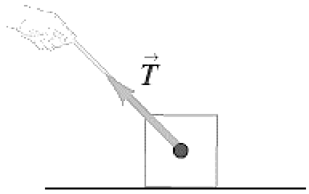
Newton's Law	Description
Newton's First Law of Motion (Law of Inertia)	Every object will continue in its state of <u>rest / uniform motion</u> in a straight line unless a <u>resultant force</u> acts on it
Newton's Second Law of Motion	When a resultant force acts on an object of constant mass, the object will <u>accelerate</u> in the <u>direction of resultant force</u> Resultant force (N) = mass (kg) × acceleration (ms^{-2})
Newton's Third Law of Motion	If body A exerts a force F_{AB} on body B, then body B exerts an equal and opposite force F_{BA} on body A

3.1 Forces

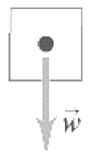
Types of forces

Type	Explanation
1. **Contact forces**	Exist between objects that are in contact
2. **Non-contact forces**	Exist between objects that are not required to be in contact

Contact forces

Types	Explanation	Figure
1. **Normal reaction**	Push exerted by surface on object pressing on it (perpendicular to surface)	
2. **Friction**	Opposes / tends to oppose motion between surfaces in contact	
3. **Tension**	Pull exerted by stretched spring / string / rope on object attached to it	

Non-contact forces

Types	Explanation	Figure
1. **Gravitational force**	Pull exerted by Earth's gravity on any object (weight)	
2. **Electric force**	Attractive / repulsive forces between electric charges	
3. **Magnetic force**	Attractive / repulsive forces between magnets	

3.2 Vector Diagrams

Vector quantities

Force: a vector – magnitude + direction

- SI unit: newton (N)
- Represented by an arrow
  - Length : proportional to magnitude of vector
  - Direction : direction of vector

Addition of vectors

Resultant vector

- Equivalent to individual vectors combined (magnitude & direction)
- Indicated by double-headed arrow
- Methods:
  1. **Parallelogram method**
  2. **Tip-to-tail method**

Forces	Resultant force	Triangle
in equilibrium	$= 0$	Form closed triangle
not in equilibrium	$\neq 0$	Do not form closed triangle

3.3 Forces and Motion

Effects of force on motion of body

Effects of force
- Body at rest moves
- Moving body increases in speed
- Moving body decreases in speed
- Moving body changes direction

Newton's First Law

Balanced forces: resultant force = 0

Motion	Figure	Forces	Resultant force
1. **At rest**		$F = W$ • Table exerts normal reaction F on book • Weight of book W	• Forces equal in magnitude, act in opposite directions • $F_R = 0$ N • $a = 0$ ms^{-2}
2. **Constant velocity**		$F = f$ • Applied force F on book • Frictional force f between book & table	

Newton's Second Law

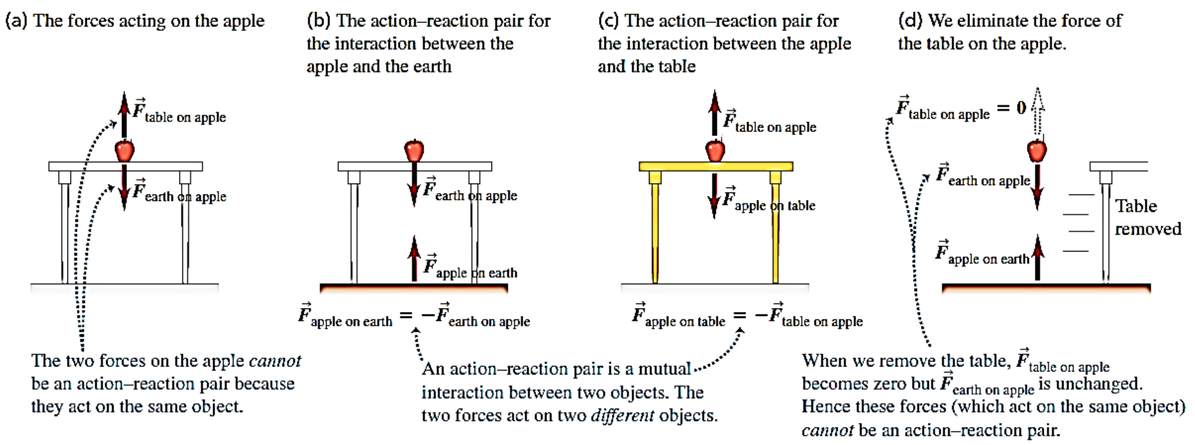
Unbalanced forces: resultant force $\neq 0 \rightarrow$ acceleration

$$\boxed{F = ma}$$

where F = resultant force (in N)
m = mass of object (in kg)
a = acceleration of object (in ms^{-2})

Newton's Third Law

Action-reaction pair:
1. Equal in magnitude
2. Same type of force
3. Act in opposite directions
4. Act on mutually opposite bodies



3.4 Friction and Its Effects

Friction affects motion

Friction (F_f)

- Opposes motion between surfaces in contact – irregularities of surfaces
- Formula:

$$F_f = \mu N$$

where f = friction force
μ = coefficient of friction
N =normal force

  - Dependent on weight of object
  - Independent of surface area

Effects of friction

Some effects:

Positive effects	Negative effects
• Walk without slipping • Slow down moving vehicles when needed	• Cars are less efficient • Moving parts (engines, motors, machines) suffer wear and tear

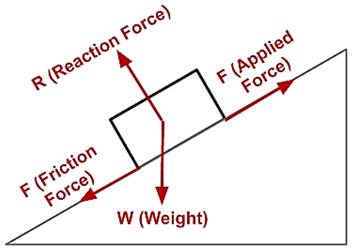
Reduce negative effects of friction

Ways	Explanation	Figure
1. **Wheels**	Contacting surfaces ✓ roll over each other ✗ drag / slide over each other	
2. **Ball bearings** (between moving parts)	Prevent moving parts rub against each other → reduce wear and tear ✓ Ball roll ✗ Flat surfaces slide against each other	
3. **Lubricants** (between moving parts of engine)	Reduce wear and tear → prolong engine life	
4. **Polish surfaces**	Remove surface irregularities → reduce friction between surfaces in contact	
5. **Air cushion**	• Hovercrafts: move on thin cushion of air • Magnetic levitation (Maglev): float above rail → electromagnetic repulsion • Travel faster	

Enhance positive effects of friction

Ways	Explanation	Figure
1. **Treads**	• Designed with treads (grooves): quickly channel water out from underneath tyres • Improve grip of tyres on wet roads → prevent skidding	
2. **Parachute**	• Larger surface area → increase air resistance significantly • Achieve safe landing	
3. **Chalk**	• Rock climbers: firm grip on rock surface • Absorb perspiration → improve grip	

3.5 Free-Body Diagrams

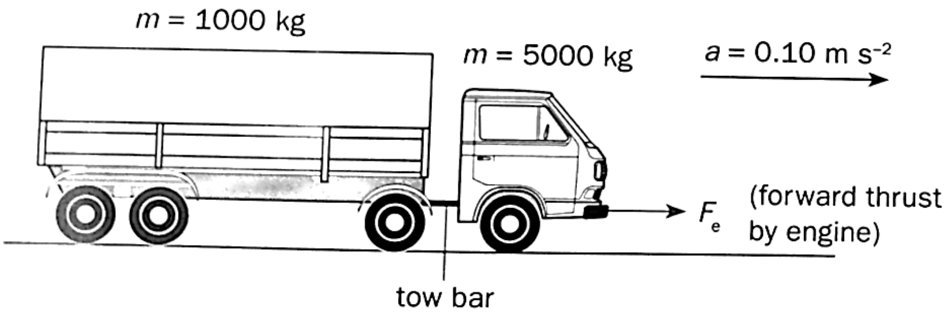


Free-body diagram: simple block diagrams with arrows to represent forces acting on individual objects

→ identify forces acting on individual objects

Example question:

A truck engine of mass 5000 kg pulls a trailer of mass 1000kg along a level track at an acceleration of 0.10 ms^{-2}. The resistances are 10 N per 1000 kg for the truck engine and 5 N per 1000 kg for the trailer.



(a) Draw a free-body diagram of the engine and the trailer.

(b) Calculate the

(i) tension in the connecting tow bar between the engine and the trailer

For the trailer, using $F = ma$, where F is the resultant force on the trailer,

$F = ma$

$T - R_t = ma$

$T = ma + R_t$

$= 1000 \text{ kg} \times 0.10 \text{ ms}^{-2} + 5 \text{ N}$

$= 105 \text{ N}$

(ii) forward thrust exerted by the engine

For the engine, using $F = ma$,

$F_e - T - R_e = ma$

$F_e = ma + T + R_e$

$= 5000 \text{ kg} \times 0.10 \text{ ms}^{-2} + 105 \text{ N} + 5 \times 10 \text{ N}$

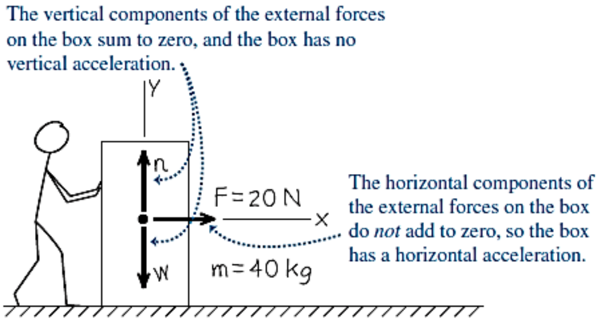
$= 655 \text{ N}$

A worker applies a constant horizontal force with magnitude 20 N to a box with mass 40 kg resting on a level, freshly waxed floor with negligible friction. What is the acceleration of the box?

IDENTIFY and SET UP This problem involves force and acceleration, so we'll use Newton's second law. That means we'll have to find the net external force acting on the box and set it equal to the mass of the box multiplied by its acceleration. In this example, the acceleration is our target variable.

In *any* problem involving forces, to find the *net* external force we must first identify all of the *individual* external forces that act on the object in question. (Remember that the net external force is the vector sum of these individual forces.) To identify these forces, we'll use the idea that two broad categories of forces act on an object like the box in **Fig. 4.17**: the *weight* of the object $\vec{w}$—that is, the downward gravitational force exerted by the earth—and *contact forces*, which are forces exerted by other objects that the object in question is touching. Two objects are touching the box—the worker's hands and the floor—and both exert contact forces on the box. The worker's hands exert a horizontal force $\vec{F}$ of magnitude 20 N. The floor exerts an upward supporting force; as in Section 4.1, we call this a *normal* force $\vec{n}$ because it acts perpendicular to the surface of contact. (Remember that "normal" is a synonym for "perpendicular." It does not mean the opposite of "abnormal"!) If friction were present, the floor would also exert a friction force on the box; we'll ignore this here, since we're told that friction is negligible. Figure 4.17 shows these three external forces that act on the box.

Figure **4.17** Our sketch for this problem.



Just as we did for the forces in Example 4.1 (Section 4.1), we'll find the vector sum of these external forces using components. That's why the second step in any problem involving forces is choosing a coordinate system for finding vector components. It's usually convenient to take one axis either along or opposite the direction of the object's acceleration, which in this case is horizontal. Hence we take the $+x$-axis to be in the direction of the applied horizontal force (which is the direction in which the box accelerates) and the $+y$-axis to be upward. In most force problems that you'll encounter (including this one), the force vectors all lie in a plane, so the z-axis isn't used.

The box doesn't move vertically, so the y-acceleration is zero: $a_y = 0$. Our target variable is the x-acceleration, a_x. We'll find it by using Newton's second law in component form, Eqs. (4.7).

EXECUTE The force $\vec{F}$ exerted by the worker has a positive x-component and zero y-component (so $F_x = F = 20\text{ N},\ F_y = 0$); the normal force $\vec{n}$ has zero x-component and an upward, positive y-component (so $n_x = 0, n_y = n$); and the weight $\vec{w}$ has zero x-component and a downward, negative y-component (so $w_x = 0, w_y = -w$). From Newton's second law, Eqs. (4.7),

$$\Sigma F_x = F + 0 + 0 = F = 20\text{ N} = ma_x$$

$$\Sigma F_y = 0 + n - w = ma_y = 0$$

From the first equation, the x-component of acceleration is

$$a_x = \frac{\Sigma F_x}{m} = \frac{20\text{ N}}{40\text{ kg}} = \frac{20\text{ kg}\cdot\text{m/s}^2}{40\text{ kg}} = 0.50\text{ m/s}^2$$

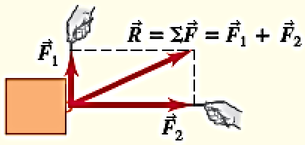
EVALUATE The net external force is constant, so the acceleration in the $+x$-direction is also constant. If we know the initial position and velocity of the box, we can find its position and velocity at any later time from the constant-acceleration equations of Chapter 2.

To determine a_x, we didn't need the y-component of Newton's second law from Eqs. (4.7), $\Sigma F_y = ma_y$. Can you use this equation to show that the magnitude n of the normal force in this situation is equal to the weight of the box?

KEYCONCEPT In problems involving forces and acceleration, first identify all of the external forces acting on an object, then choose a coordinate system. Find the vector sum of the external forces, and then set it equal to the mass of the object times the acceleration.

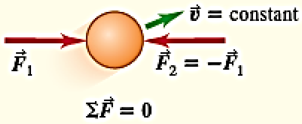
Force as a vector: Force is a quantitative measure of the interaction between two objects. It is a vector quantity. When several external forces act on an object, the effect on its motion is the same as if a single force, equal to the vector sum (resultant) of the forces, acts on the object. (See Example 4.1.)

$$\vec{R} = \sum\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots \quad (4.1)$$



The net external force on an object and Newton's first law: Newton's first law states that when the vector sum of all external forces acting on a object (the *net external force*) is zero, the object is in equilibrium and has zero acceleration. If the object is initially at rest, it remains at rest; if it is initially in motion, it continues to move with constant velocity. This law is valid in inertial frames of reference only. (See Examples 4.2 and 4.3.)

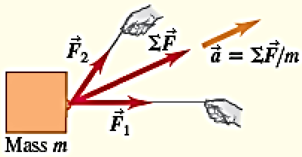
$$\sum\vec{F} = 0 \quad (4.3)$$



Mass, acceleration, and Newton's second law: The inertial properties of an object are characterized by its *mass*. Newton's second law states that the acceleration of an object under the action of a given set of external forces is directly proportional to the vector sum of the forces (the *net force*) and inversely proportional to the mass of the object. Like Newton's first law, this law is valid in inertial frames of reference only. In SI units, the unit of force is the newton (N), equal to 1 kg · m/s^2. (See Examples 4.4 and 4.5.)

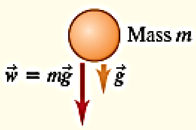
$$\sum\vec{F} = m\vec{a} \quad (4.6)$$

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z \quad (4.7)$$



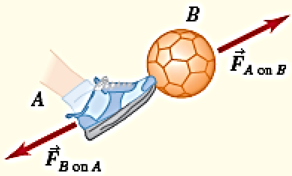
Weight: The weight $\vec{w}$ of an object is the gravitational force exerted on it by the earth. Weight is a vector quantity. The magnitude of the weight of an object at any specific location is equal to the product of its mass m and the magnitude of the acceleration due to gravity g at that location. The weight of an object depends on its location; its mass does not. (See Examples 4.6 and 4.7.)

$$w = mg \quad (4.8)$$



Newton's third law and action–reaction pairs: Newton's third law states that when two objects interact, they exert forces on each other that are equal in magnitude and opposite in direction. These forces are called action and reaction forces. Each of these two forces acts on only one of the two objects; they never act on the same object. (See Examples 4.8–4.11.)

$$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A} \quad (4.10)$$



<u>Typical questions</u>

Multiple choice questions

1. When a horse pulls a wagon, the force that causes the horse to move forward is:
   A the force it exerts on the ground
   B the force it exerts on the wagon
   C the force the ground exerts on it
   D the force the wagon exerts on it

2. A coin is tossed vertically up in the air. It first rises and then falls. As the coin passes through its highest point, the net force on it:
   A becomes zero
   B acts downwards and reaches a maximum value
   C acts downwards and reaches a minimum value
   D acts downwards and remains constant

3. A helicopter flies horizontally with constant velocity. The horizontal net force acting on it is:
   A parallel to the velocity
   B vertically upward
   C vertically downward
   D zero

4. A constant force is exerted on a cart that is initially at rest on a track. Friction between the cart and the track is negligible. The force acts for a short time interval and gives the cart a certain final speed. To reach the same final speed with a force that is only half as big, the force must be exerted on the cart for a time interval
   A four times as long as for the stronger force
   B twice as long as for the stronger force
   C half as long as for the stronger force
   D a quarter as long as for the stronger force

5. A constant force is exerted for a short time interval on a cart that is initially at rest on frictionless track. This force gives the cart a certain final speed. The same force is exerted for the same length of time on another cart, also initially at rest, that has twice the mass of the first one. The final speed of the heavier cart is
   A one-fourth that of the lighter cart
   B half that of the lighter cart
   C the same as that of the lighter cart
   D double that of the lighter cart

6. A rocket of mass 10000 kg on the surface of the earth accelerated upward at a rate of 4.0 m/s2. The force provided by the rocket engines must be:
   A 40000 N
   B 100000 N
   C 140000 N
   D 160000 N

7. A big ship crashes into a small canoe. During the collision, the force that the ship exerts on the canoe is:
   A greater than the force the canoe exerts on the ship
   B equal to the force the canoe exerts on the ship
   C less than the force the canoe exerts on the ship
   D is related to the force on the canoe in a way that depends on the nature of the collision

8. A mass of 30 kg on a smooth horizontal table is tied to a cord running along the table over a frictionless pulley mounted at the edge of the table. A 10 kg mass is attached to the other end of the cord. When the two masses are allowed to move freely, the tension in the cord is:
   A 300 N
   B 150 N
   C 100 N
   D 75 N

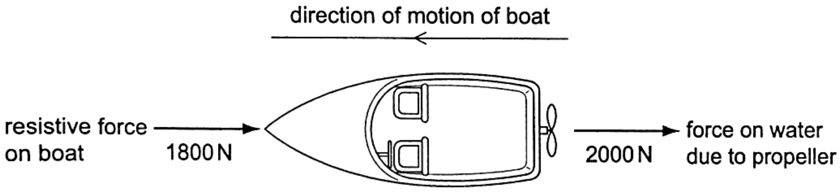
9. A stationary book sits on a table. Newton's third law states that "to every action there is an equal and opposite reaction". The reaction to the weight of the book is the force that:
   A earth exerts on the book
   B book exerts on the table
   C table exerts on the book
   D book exerts on the earth

10. When his parachute is fully opened, a parachutist falls towards the ground at constant speed.
    Under these conditions, which statement is correct? (2011 P1 Q5)
    A There are no forces acting on the parachutist.
    B The upward force on the parachute is equal to the weight of the parachutist.
    C The upward force on the parachute is greater than the weight of the parachutist.
    D The upward force on the parachute is less than the weight of the parachutist.

11. A swimmer of mass 60 kg dives into a deep pool from a high diving platform. She enters the water at high speed and then decelerates at a rate of 16 m/s^2 as she travels vertically downwards in the water. The gravitational field strength is 10 N/kg.
What is the swimmer's weight and the size of the resultant force acting on her as she decelerates? (2013 P1 Q6)

	swimmer's weight	resultant force
A	60 kg	960 N
B	60 kg	9600 N
C	600 N	960 N
D	600 N	9600 N

12. The propeller on a boat pushes water backwards with a force of 2000 N. The boat moves through the water against a total resistive force of 1800 N.



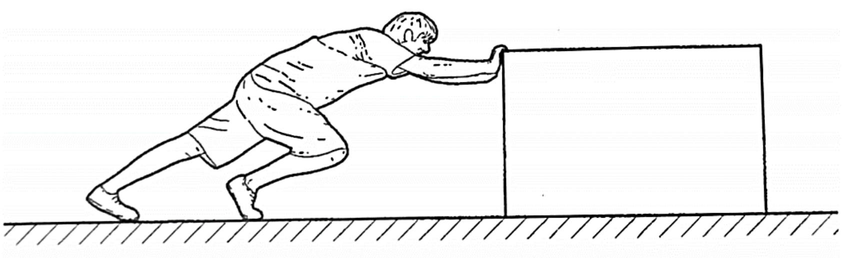
What is the forward force on the propeller due to the water? (2015 P1 Q6)

A 200 N
B 1800 N
C 2000 N
D 3800 N

13. A car of mass 800 kg has a forward acceleration of 2.5 m/s^2. A frictional force of 1200 N opposes the motion of the car.
What is the driving force due to the engine of the car? (2015 P1 Q7)

A 800 N
B 1200 N
C 2000 N
D 3200 N

14. A man pushes a heavy box along the ground.

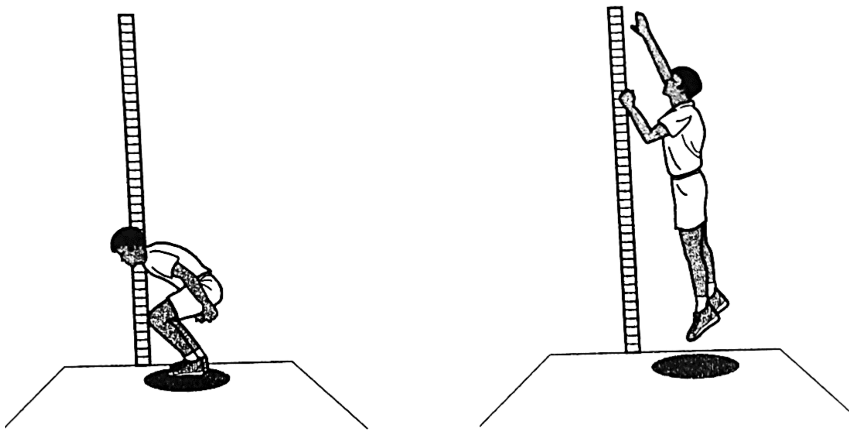


A force acts between the man's hands and the box.
Another force acts between the man's feet and the floor.
In which direction do these forces act on the man? (2018 P1 Q12)

	force on man's hands	force on man's feet
A	towards the left	towards the left
B	towards the left	towards the right
C	towards the right	towards the left
D	towards the right	towards the right

Structured questions

1. A man jumps vertically upwards. The figure below shows the man in two positions. (2016 P2A Q2)



(a) On the figure above, draw and label the forces acting on the man in each position. You may ignore air resistance. [2]

(b) The man has a mass of 60 kg. He leaves the ground with a speed of 1.5 m/s. The time taken for the man to rise from his lowest position until his feet just leave the ground is 0.50 s.

(i) Determine the average resultant force on the man during this time. [2]

$a = \frac{v-u}{t} = \frac{1.5 - 0}{0.50} = 3.0 \text{ ms}^{-2}$

$F_R = ma$

$= 60 \text{ kg} \times 3.0 \text{ ms}^{-2}$

$= 180 \text{ N}$

(ii) The man lands and stands at rest with both feet on the ground.
The area of contact between his shows and the ground is 500 cm^2.
The gravitational field strength g is 10 N/kg.
Determine the pressure exerted by the man on the ground. Give your answer in Pa. [2]

Area = 500 cm^2 = 0.05 m^2

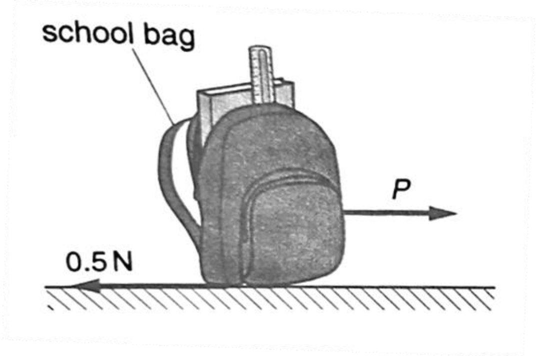
$W = mg$ = (60)(10) = 600 N

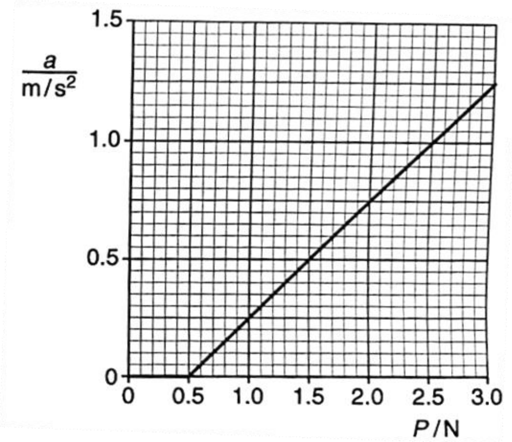
$P = \frac{F}{A} = \frac{600}{0.05}$ = 12000 Pa

(iii) The pressure exerted on the ground during the first 0.50 s of the jump is larger than the value calculated in (ii). Explain why. [1]

The force exerted by the man on the ground is greater than just the weight, as he accelerates to push himself off the ground.

2. A student pulls a school bag along a table with a force P, as shown in the left figure below.
When the bag is moving there is a frictional force of 0.5 N acting as shown.
The right figure below shows how the acceleration a of the bag varies with P. (2014 P2B Q10)





(a) Explain why P must be greater than 0.5 N for the bag to accelerate. [1]
When force P is greater than 0.5 N, there is a resultant force acting on the school bag in the direction of P, thus causing the bag to accelerate.

(b) (i) State the equation relating to resultant force F, mass m of the bag and acceleration a. [1]
$F = ma$

(ii) Using the right figure above, calculate the value of m. [2]
From the graph, when force P is 3.0 N, the acceleration is 1.25 m/s^2.
$F_R = 3.0 - 0.5 = 2.5$ N
$F_R = ma$
$m = \frac{2.5}{1.25} = 2$ kg

(c) The force P is held constant at 2.0 N.
The bag accelerates from rest at time $t = 0$ until $t = 3.0$ s.
(i) Determine the speed at $t = 3.0$ s. [2]
When force P is 2.0 N, the acceleration is 0.75 m/s^2. This means that the school bag has an increase in velocity of 0.75 m/s every second.
After 3 seconds, the increase in velocity = $0.75 \times 3 = 2.25$ m/s.
Since the school bag was initially at rest, the speed of the school bag is now 2.25 m/s.

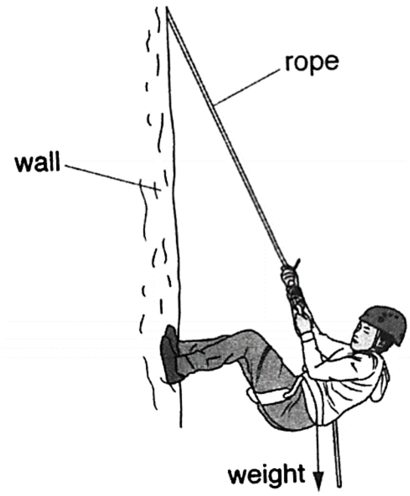
(ii) At $t = 3.0$ s, the force P is reduced to 0.5N.
Using Newton's laws, explain what happens to the bag. [2]
When the force is reduced to 0.5 N, force P is balanced with the frictional force. Since the forces are balanced (resultant force = 0 N), by Newton's first law, the school bag will continue to stay in motion at a constant speed of 2.25 m/s in a straight line.

(d) The force P and the frictional force in the left figure above are **not** a Newton's Third Law action-reaction pair. Describe the other force that is the part of the action-reaction pair with P, and state the body on which it acts. [2]
When the student pulls on the bag with force P, the bag pulls back on the student with an equal and opposite reaction force. Together, these two forces form an action-reaction pair.

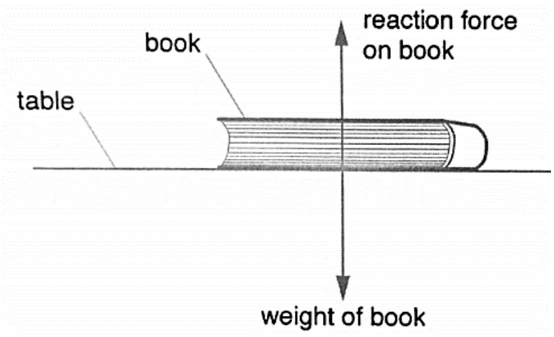
3. (a) Describe the effects of unbalanced forces on the motion of an object. [2]

Unbalanced forces cause an object to accelerate (or decelerate) or change its direction of motion, with the resultant force, $F_R = ma$.

(b) The figure shows a climber supported by a rope on a vertical wall.
The weight of the climber is shown by the labelled arrow.
On the figure below, draw arrows to show the directions of the other forces acting on the climber.
State the names of the forces next to the arrows that you draw. [2]



(c) A student is asked to draw an example of an action-reaction pair of forces, as described by Newton's third law. The figure below shows the diagram drawn by the student.



The teacher states that the figure above does **not** show an action-reaction pair of forces. She adds that forces in an action-reaction pair have similarities and differences.

(i) State two ways in which the two forces in an action-reaction pair are identical. [2]

1 The two forces are of the same type.

2 The magnitudes of the two forces are equal.

(ii) State two ways in which the two forces in an action-reaction pair are different. [2]

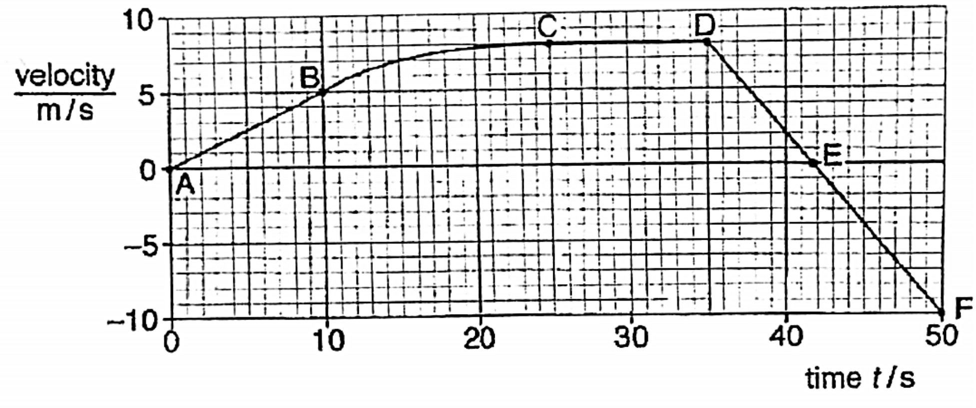
1 The two forces are opposite in direction.

2 The two forces act on two different objects.

(iii) In the figure above, the Earth exerts a force of 10 N on the book resting on the table. This is one force in the action-reaction pair.
Describe the other force in the pair. [2]

The book exerts a 10 N force on the Earth, in the opposite direction.

4. The figure below shows the velocity-time graph for a car initially travelling forward in a horizontal straight line. (5059/N17/2/10)



(a) Describe the motion of the car.

(i) between A and B [1]

constant acceleration

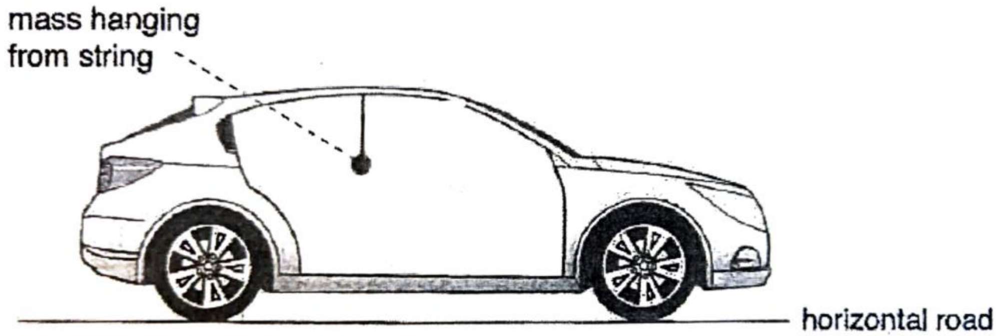
(ii) between B and C [1]

decreasing acceleration (to zero)

(iii) between E and F [1]

constant deceleration

(b) A small mass hangs freely from a string suspended inside the car. The figure shows the position of the string and the mass at time t = 30.0 s.



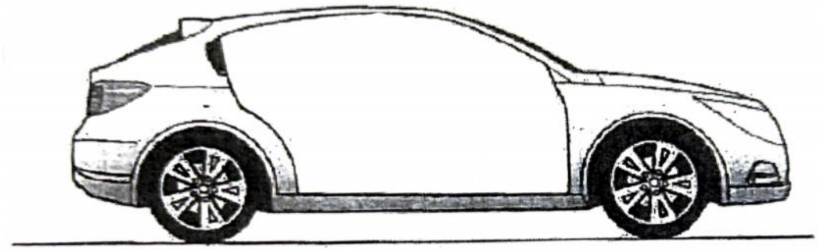
(i) On the figure above, draw and label the forces acting on the mass. [2]

(ii) Explain, using Newton's laws of motion, why the string is vertical, even though the car is moving. [2]

Newton's first law states that an object will continue in its state of rest / uniform motion in a straight line unless a resultant force acts on it.

At t = 30.0 s, the car and the mass move at constant velocity, hence the horizontal resultant force on the mas is zero and the string is vertical.

(iii) On the diagram below, sketch the position of the mass and the string at time t = 5.0 s. [1]



(iv) Using Newton's laws of motion, explain your answer in the figure above. [2]

Newton's second law states that when a resultant force acts on an object of constant mass, the object will accelerate in the direction of resultant force.

At t = 5.0 s, the car and the mass move with constant forward acceleration, hence there is a forward resultant force on the mass, which is equal to the resultant of the tension and the weight. Therefore, the string must be inclined in the position shown to provide for this forward force.

5. A box of books with mass 55 kg rests on the level floor of the campus bookstore. The floor is freshly waxed and has negligible friction. A bookstore worker applies a constant horizontal force with magnitude 25 N to the box. What is the magnitude of the acceleration of the box?

6. A block of cheese of mass 2.0 kg sits on a freshly waxed, essentially frictionless table. You apply a constant horizontal force of 0.50 N to the cheese.
   (a) Name the three external forces that act on the cheese and what exerts each force.
   (b) What is the magnitude of the acceleration of the cheese?

7. In a game of ice hockey, you use a hockey stick to hit a puck of mass 0.16 kg that slides on essentially frictionless ice. During the hit you exert a constant horizontal force on the puck that gives it an acceleration of 75 m/s^2 for a fraction of a second.
   (a) During the hit, what is the magnitude of the horizontal force that you exert on the puck?
   (b) How does the magnitude of the normal force due to the ice compare to the weight of the puck?

8. Two dogs pull horizontally on ropes attached to a post; the angle between the ropes is 60.0. If Rover exerts a force of 270 N and Fido exerts a force of 300 N, find the magnitude of the resultant force and the angle it makes with Rover's rope.