Chapter 2 – Kinematics

Subject content

Content
- Speed, velocity and acceleration
- Graphical analysis of motion
- Free-fall
- Effect of air resistance

Learning outcomes

(a) state what is meant by speed and velocity
(b) calculate average speed using distance travelled / time taken
(c) state what is meant by uniform acceleration and calculate the value of an acceleration using change in velocity / time taken
(d) interpret given examples of non-uniform acceleration
(e) plot and interpret a displacement-time graph and a velocity-time graph
(f) deduce from the shape of a displacement-time graph when a body is:
  (i) at rest
  (ii) moving with uniform velocity
  (iii) moving with non-uniform velocity
(g) deduce from the shape of a velocity-time graph when a body is:
  (i) at rest
  (ii) moving with uniform velocity
  (iii) moving with uniform acceleration
  (iv) moving with non-uniform acceleration
(h) calculate the area under a velocity-time graph to determine the displacement travelled for motion with uniform velocity or uniform acceleration
(i) state that the acceleration of free fall for a body near to the Earth is constant and is approximately 10 m / s^2
(j) describe the motion of bodies with constant weight falling with or without air resistance, including reference to terminal velocity

Definitions

Phrase	Definition
Distance	Total length travelled by object irrespective of the direction of motion
Displacement	Distance travelled by object in a specified direction
Speed	Rate of change of distance (distance travelled per unit time)
Velocity	Rate of change of displacement (distance travelled in a specified direction per unit time)
Acceleration	Rate of change of velocity

Formulae

$v = \frac{s}{t}$	$a = \frac{v - u}{t}$	$s = \frac{1}{2}(u + v) \times t$

2.1 Distance and Displacement

Scalars & vectors

Physical quantity	Properties	Examples
1. **<u>Scalar</u> quantities**	✓ magnitude ✗ direction	(a) Distance (b) Speed (c) Mass (d) Energy (e) Time
2. **<u>Vector</u> quantities**	✓ magnitude ✓ direction	(a) Displacement (b) Velocity (c) Acceleration (d) Force

<u>Difference between distance and displacement</u>

Distance

Total length travelled by object irrespective of direction of motion

Displacement (shortest distance) – need to state direction

Distance travelled by object in a specified direction

2.2 Speed, Velocity and Acceleration

<u>Speed</u>

Speed

Distance travelled per unit time
OR
Rate of change of speed

Speed:

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

Average speed:

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}$$

<u>Velocity</u>

Velocity

Displacement travelled per unit time
OR
Rate of change of displacement

Velocity:

$$\text{velocity} = \frac{\text{displacement}}{\text{time}}$$

Average velocity:

$$\text{average velocity} = \frac{\text{total displacement}}{\text{total time}}$$

Instantaneous speed: speed at a particular instant

<u>Acceleration</u>

Acceleration

Rate of change of velocity

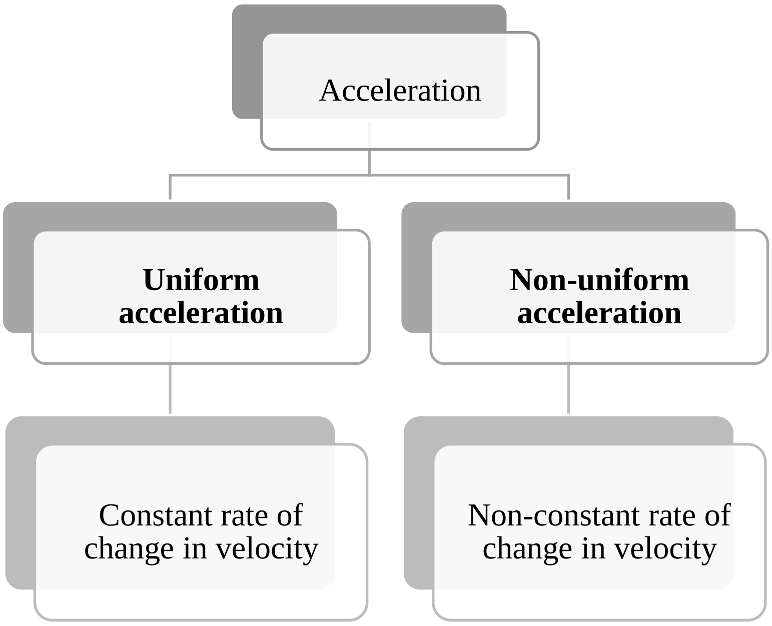
Acceleration:

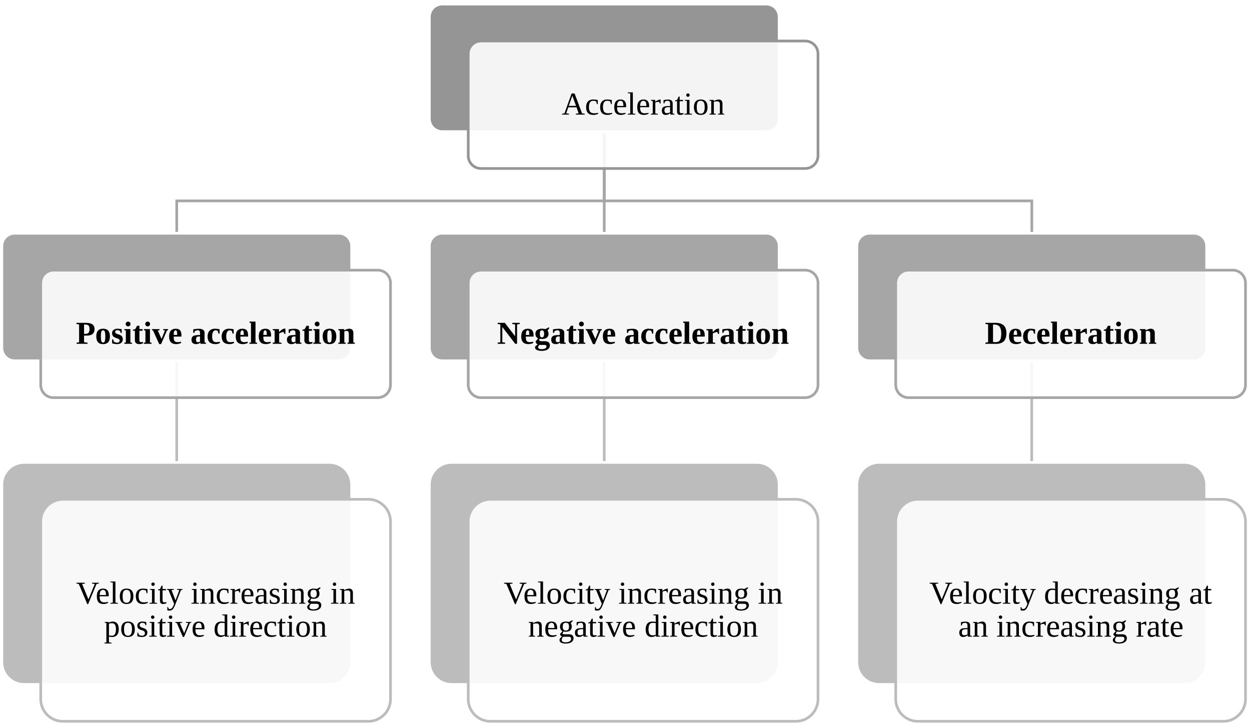
$$a = \frac{\Delta v}{\Delta t} = \frac{v - u}{t_v - t_u}$$

where u = initial velocity
v = final velocity
t_u = time at which object is at initial velocity u (in s)
t_v = time at which object is at final velocity v (in s)
Δv = change in velocity (in ms^{-1})
Δt = time interval between t_u and t_v (in s)

Objects in motion undergo acceleration when:

1. Change in speed
2. Change in direction





Scalar quantity	Vector quantity	Equation in symbols	SI unit
Distance	**Displacement**	s	m
Speed	**Velocity**	$v = \frac{\Delta s}{\Delta t}$	ms^{-1}
Average speed	**Average velocity**	average $v = \frac{\text{total } s}{\text{total } t}$	ms^{-1}
	Acceleration	$a = \frac{v - u}{\Delta t}$	ms^{-2}

2.3 Graphs of Motion

Graphs

	s-*t* graph	*v*-*t* graph
Distance / displacement	Vertical axis	Area under graph
Speed / velocity	Gradient	Vertical axis
Acceleration	Second derivative of displacement with respect to time – $\frac{d^2s}{dt^2}$	Gradient

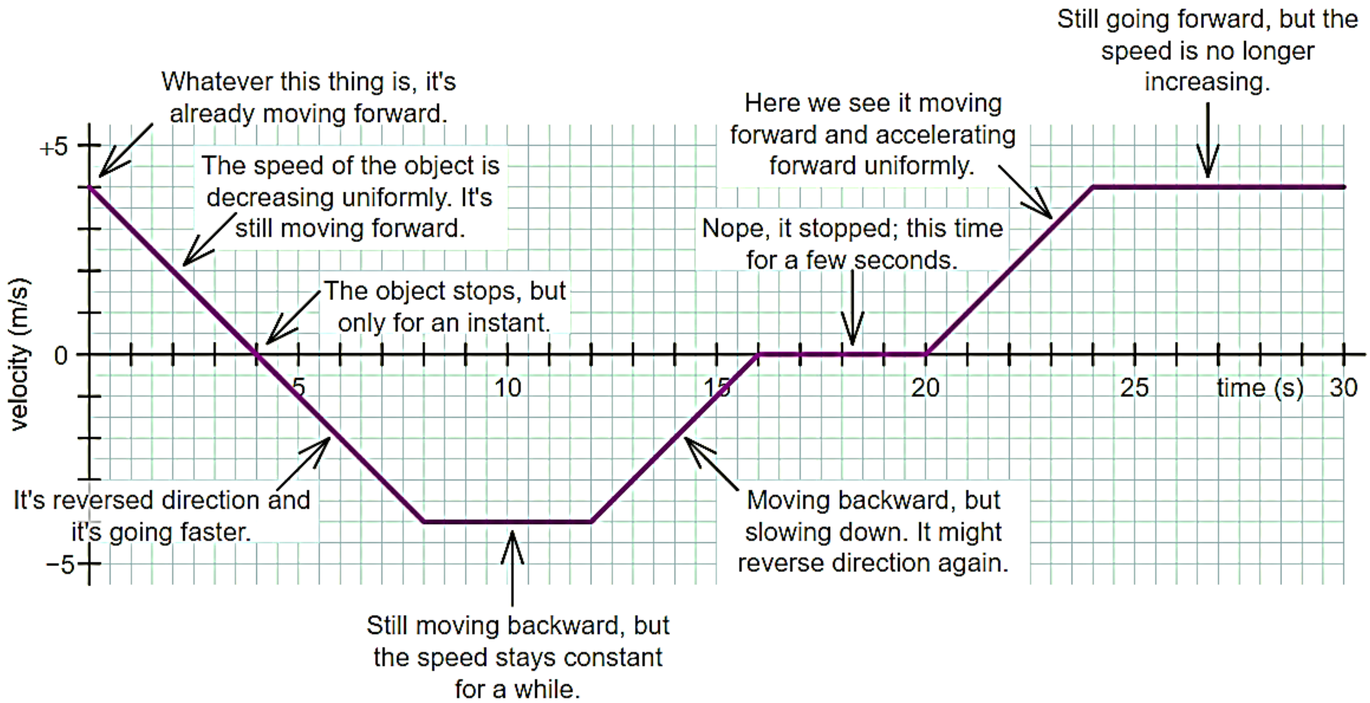
Deriving acceleration for displacement-time graph:

s = distance

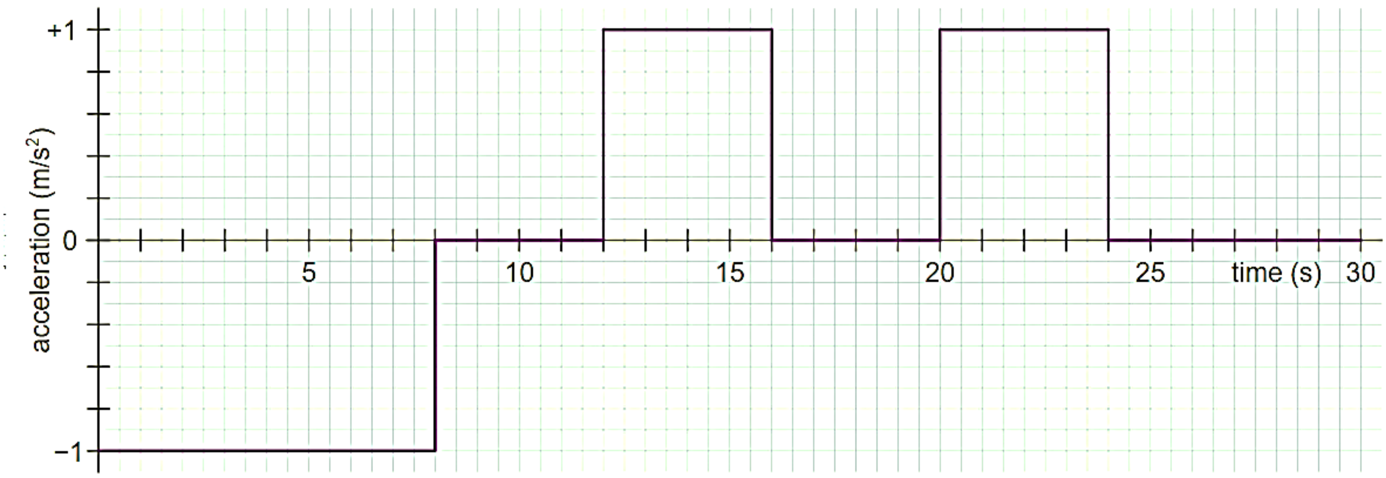
v = velocity = $\frac{ds}{dt}$

a = acceleration = $\frac{dv}{dt}$

	s-t graph	v-t graph
y-intercept	initial displacement	Initial velocity
Slope of tangent	Instantaneous velocity	Instantaneous acceleration
Positive slope	Motion in positive direction	Acceleration in positive direction
Negative slope	Motion in negative direction	Acceleration in negative direction
Zero slope	Not moving (at rest)	Not accelerating (constant velocity)
Straight	Constant velocity	Constant acceleration
Curved	Changing velocity	Changing acceleration
Area under graph	-	Total displacement
Curves coincide	Objects have same displacement	Objects have same velocity
Stopped when	Horizontal	Crosses time axis
Uniform acceleration	Parabolic	Straight



Whatever this thing is, it's already moving forward.
The speed of the object is decreasing uniformly. It's still moving forward.
The object stops, but only for an instant.
It's reversed direction and it's going faster.
Still moving backward, but the speed stays constant for a while.
Moving backward, but slowing down. It might reverse direction again.
Nope, it stopped; this time for a few seconds.
Here we see it moving forward and accelerating forward uniformly.
Still going forward, but the speed is no longer increasing.
velocity (m/s)
+5
0
−5
5
10
15
20
25
time (s)
30

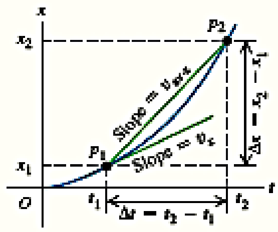


acceleration (m/s²)
+1
0
−1
5
10
15
20
25
time (s)
30

Straight-line motion, average and instantaneous x-velocity: When a particle moves along a straight line, we describe its position with respect to an origin O by means of a coordinate such as x. The particle's average x-velocity $v_{av\text{-}x}$ during a time interval $\Delta t = t_2 - t_1$ is equal to its displacement $\Delta x = x_2 - x_1$ divided by Δt. The instantaneous x-velocity v_x at any time t is equal to the average x-velocity over the time interval from t to $t + \Delta t$ in the limit that Δt goes to zero. Equivalently, v_x is the derivative of the position function with respect to time. (See Example 2.1.)

$$v_{av\text{-}x} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \qquad (2.2)$$

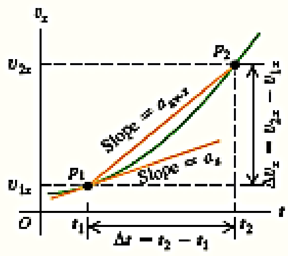
$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad (2.3)$$



Average and instantaneous x-acceleration: The average x-acceleration $a_{av\text{-}x}$ during a time interval Δt is equal to the change in velocity $\Delta v_x = v_{2x} - v_{1x}$ during that time interval divided by Δt. The instantaneous x-acceleration a_x is the limit of $a_{av\text{-}x}$ as Δt goes to zero, or the derivative of v_x with respect to t. (See Examples 2.2 and 2.3.)

$$a_{av\text{-}x} = \frac{\Delta v_x}{\Delta t} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} \qquad (2.4)$$

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \qquad (2.5)$$



Straight-line motion with constant acceleration: When the x-acceleration is constant, four equations relate the position x and the x-velocity v_x at any time t to the initial position x_0, the initial x-velocity v_{0x} (both measured at time $t = 0$), and the x-acceleration a_x. (See Examples 2.4 and 2.5.)

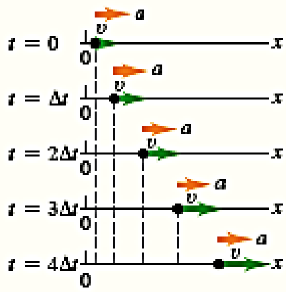
Constant x-acceleration only:

$$v_x = v_{0x} + a_x t \qquad (2.8)$$

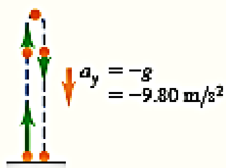
$$x = x_0 + v_{0x} t + \tfrac{1}{2} a_x t^2 \qquad (2.12)$$

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0) \qquad (2.13)$$

$$x - x_0 = \tfrac{1}{2}(v_{0x} + v_x)t \qquad (2.14)$$



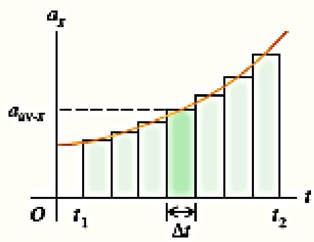
Freely falling objects: Free fall (vertical motion without air resistance, so only gravity affects the motion) is a case of motion with constant acceleration. The magnitude of the acceleration due to gravity is a positive quantity, g. The acceleration of an object in free fall is always downward. (See Examples 2.6–2.8.)



Straight-line motion with varying acceleration: When the acceleration is not constant but is a known function of time, we can find the velocity and position as functions of time by integrating the acceleration function. (See Example 2.9.)

$$v_x = v_{0x} + \int_0^t a_x\,dt \qquad (2.17)$$

$$x = x_0 + \int_0^t v_x\,dt \qquad (2.18)$$



Total displacement = area under velocity-time graph

$$s = \frac{1}{2}(u + v) \times t$$

Speed-time graph against velocity-time graph

Upward then downward motion of ball

Speed-time graph	Velocity-time graph
	

Downward then upward motion of ball

Speed-time graph	Velocity-time graph
	

2.4 Acceleration due to Gravity

Galileo's discovery

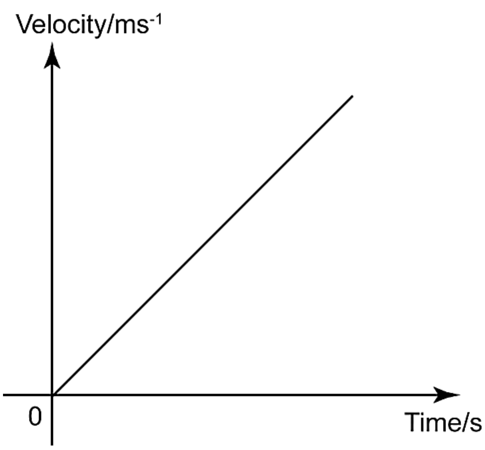
All objects, regardless of mass or size, fall at the **same acceleration** due to the Earth's gravity

Acceleration due to gravity (**g**): 9.8 ms^{-2} (≈ **10 ms^{-2}**)

Objects falling without air resistance

Free fall: only force acting is **weight** (vacuum – no air resistance)

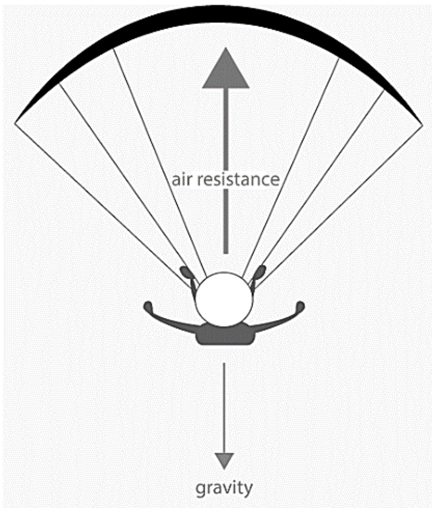
- Velocity under gravity : increase by 10 ms^{-1} every second
  Acceleration : constant at 10 ms^{-2}
- Direction of motion : downward
- Acceleration : does not depend on mass / size



Objects falling with air resistance

Air resistance: frictional force acted on object by air

1. Opposes motion of moving objects
2. Increases with speed of objects
3. Increases with surface area / size of objects
4. Increases with density of air
5. Non-linear (exponential)



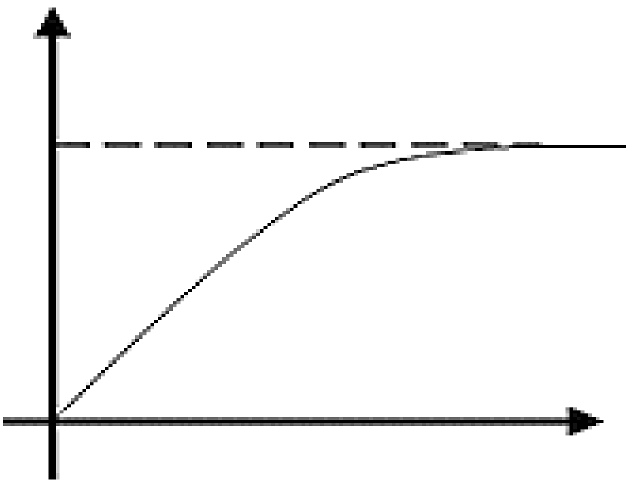
Air resistance pushes upwards against the open parachute to counter weight of parachutist

Terminal velocity: **constant velocity** that object travels when $W = R$

Weight	Terminal velocity
Heavier	• More air resistance acting against it • Higher terminal velocity, reached at shorter time
Lighter	• Less air resistance acting against it • Lower terminal velocity, reached at longer time

Object falling with air resistance

Time interval	Forces involved	Changes	
		Velocity	Acceleration (ms^{-2})
$t = 0$ to $t = t_1$	$\boldsymbol{W > R_1}$	increases from 0 to v_1	**10**
$t = t_1$ to $t = t_2$	$\boldsymbol{W > R_2}$ ($R_2 > R_1$)	increases from v_1 to v_2	**< 10**
$t = t_2$ to $t = t_3$	$\boldsymbol{W = R_3}$ ($R_3 > R_2$)	constant	**0**



For falling objects, acceleration $= \dfrac{\text{resultant force (N)}}{\text{mass (kg)}}$

$$a = \frac{F}{m} = \frac{\text{weight} - \text{air resistance}}{\text{mass}}$$
$$= \frac{W - R}{m}$$

Skydiving

Two forces acting on skydiver

1. **Weight** – W (downward force)
2. **Air resistance** – R (upward force)

Factors affecting air resistance

<table>
<tr><th>Factor</th><th>Air resistance</th><th colspan="2">Explanation</th></tr>
<tr><td>1. Speed</td><td>increases with increasing speed</td><td colspan="2">• W > R: fall with acceleration of 10 ms^{-2}
• Gain speed, R increases until equals W
• Falls at terminal velocity (acceleration = 0 ms^{-2})</td></tr>
<tr><td rowspan="3">2. Surface area</td><td rowspan="3">increases with smaller surface area</td><td>Spread-eagle position</td><td>Fall head / feet first</td></tr>
<tr><td>maximise surface area</td><td>minimise surface area</td></tr>
<tr><td>• Achieve lower terminal velocity
• Stay in air longer</td><td>• Achieve higher terminal velocity
• Stay in air shorter</td></tr>
</table>

Effect of parachute:

- Increase surface area → increase air resistance
- Air resistance overwhelms downward force of gravity. Net force and acceleration on falling skydiver is upward. An upward net force on a downward falling object causes skydiver to slow down.
- As his speed decreases, air resistance decreases until skydiver reaches a lower terminal velocity.

Typical questions

Multiple choice questions

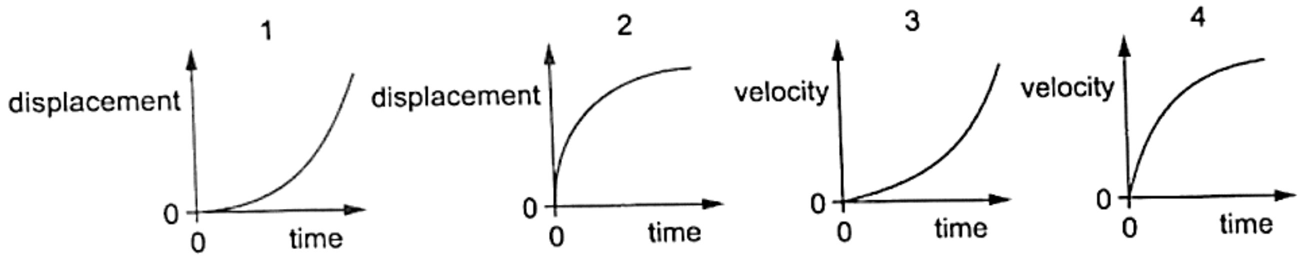
1. An object falls through a vacuum. Which row describes the acceleration and the velocity of the object? (2012 P1 Q4)

	acceleration	velocity
A	constant	constant
B	constant	increasing
C	increasing	constant
D	increasing	increasing

2. A stone falling through the air has reached terminal velocity. Which row shows the acceleration and the velocity of the stone? (2014 P1 Q5)

	acceleration of the stone	velocity of the stone
A	zero	constant
B	zero	increasing
C	10 m/s^2	constant
D	10 m/s^2	increasing

3. A box is dropped from the top of a tall cliff. As the box falls, air resistance has a noticeable effect on its motion. Graphs 1 and 2 are displacement-time graphs. Graphs 3 and 4 are velocity-time graphs.



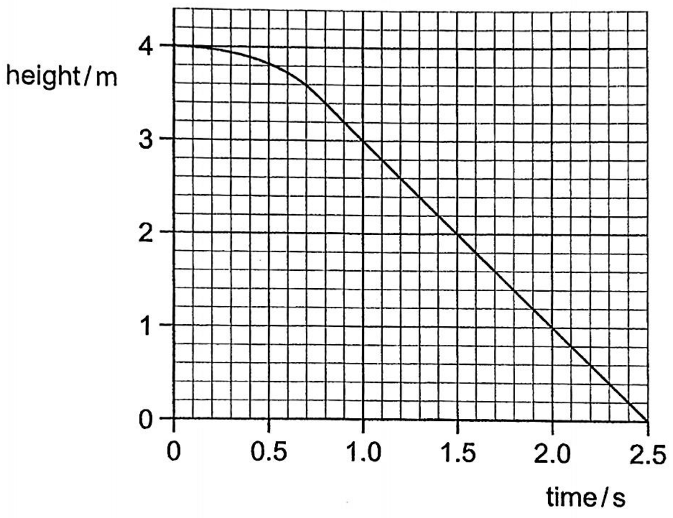
Which graphs describe the motion of the box? (2015 P1 Q5)

A 1 and 3
B 1 and 4
C 2 and 3
D 2 and 4

4. Two objects have the same size and shape but one is heavier than the other. They are each dropped from rest in air.
Comparing the two objects, the heavier object has (2016 P1 Q3)

A higher initial acceleration and higher terminal velocity
B higher initial acceleration and same terminal velocity
C same initial acceleration and higher terminal velocity
D same initial acceleration and same terminal velocity

5. The graph shows how the height of a falling object changes with time.



What is the terminal velocity of the object? (2017 P1 Q3)

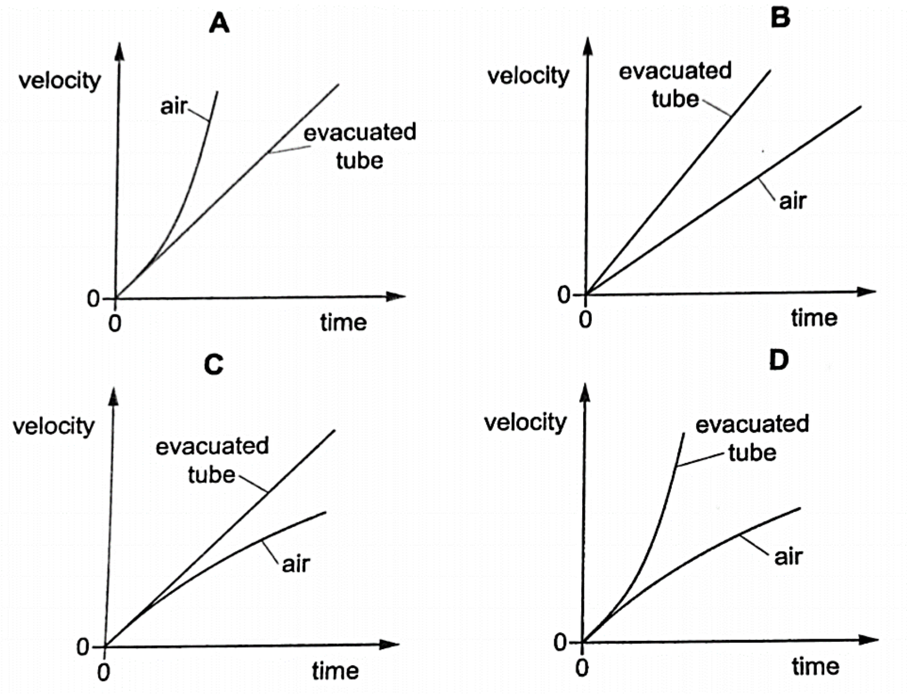
A 1.0 m/s

B 1.3 m/s

C 1.6 m/s

D 2.0 m/s

6. A ball falls towards the ground, first in air and then later in an evacuated tube, which has most of the air in it removed.
What graph shows how the velocity of the ball changes with time in these cases? (2018 P1 Q5)



7. A train travels north at a velocity of 25 m/s along a straight, horizontal track.
At time t = 5.0 s, its velocity starts to change and its acceleration is -2.0 m/s^2.
How is the train moving at time t = 15.0 s? (2018 P1 Q7)

A travelling north with decreasing speed

B travelling north with increasing speed

C travelling south with decreasing speed

D travelling south with increasing speed

8. A heavy object is dropped from an aeroplane. The velocity of the object is measured at two points before it hits the ground. The velocities are found to be the same.
   What is the explanation for this? (2019 P1 Q6)
   A All objects fall at constant velocity.
   B The force on the object due to air resistance is equal to the acceleration of free fall g.
   C The force on the object due to air resistance is equal to the object's weight.
   D The force on the object due to air resistance is larger than the object's weight.

Structured questions

1. An arrow leaves a bow with a speed of 42 m/s. Its velocity is reduced to 34 m/s by the time it hits the target within 2.4 s. What is its average deceleration while travelling in the air?

$$a = \frac{v-u}{\Delta t}$$

$$= \frac{34-42}{2.4}$$

$$= -3.33 \text{ ms}^{-2}$$

∴ Deceleration = 3.33 ms^{-2}

2. Harry and a group of friends are driving north along an expressway. As they approach a bend, the driver rounds a curve without changing speed and continues driving east.
   (a) Does the velocity change? Explain your answer.
   Yes, the velocity changes.
   As the driver rounds the curve, the direction of motion changes.

   (b) Are Harry and his friends accelerating? Explain your answer.
   Yes, they are accelerating.
   Since there is a change in velocity, they are accelerating, which is defined as the rate of change of velocity.

3. The table shows the speed of three lorries at several values of time t.

Lorry	Speed at t = 0 s	Speed at t = 5 s	Speed at t = 10 s	Speed at t = 20 s	Speed at t = 40 s
A	0 m/s	4.0 m/s	8.0 m/s	16.0 m/s	32.0 m/s
B	0 m/s	6.0 m/s	12.0 m/s	18.0 m/s	24.0 m/s
C	0 m/s	8.0 m/s	16.0 m/s	20.0 m/s	20.0 m/s

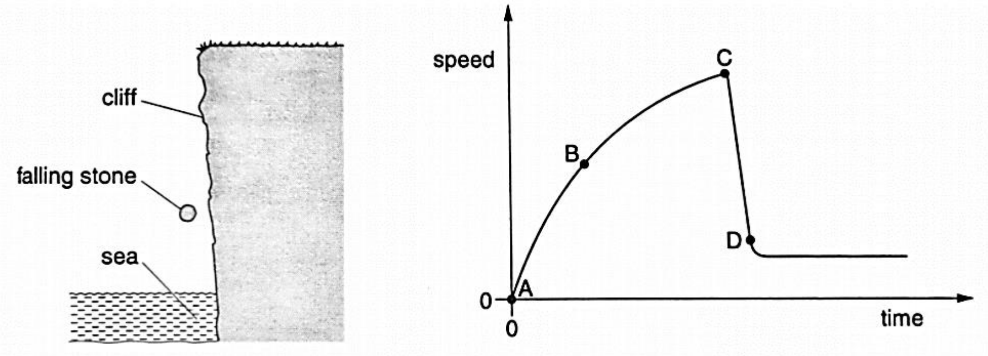
   (a) State which lorry has the greatest initial acceleration. Explain your answer.
   Lorry C.
   The change in speed for the same time interval (from t = 0 s to t = 5 s) is 8.0 m/s which is the largest, causing the greatest initial acceleration.

   (b) State which lorry has a uniform acceleration. Explain your answer.
   Lorry A.
   Change in speed per unit time is constant at 0.80 ms^{-2}.

4. Describe the motion of the object in the following sections.

A–B : increasing acceleration
B–C : constant acceleration
C–D : decreasing acceleration
D–E : increasing deceleration
E–F : constant deceleration
F–G : decreasing deceleration

5. A stone falls from the top of a cliff into the sea, as shown in the left figure below. The speed-time graph for the stone is shown in the right figure below. (2013 P2A Q1)



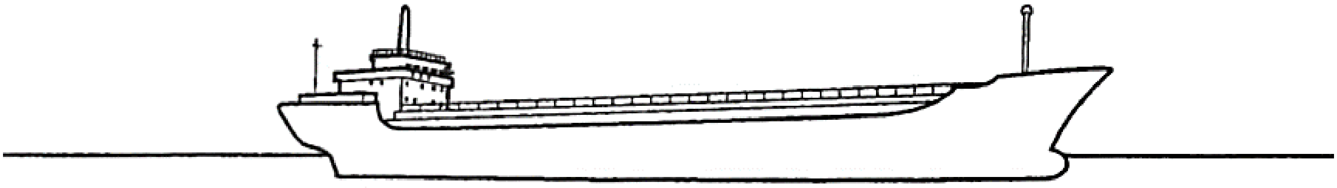
(a) State how acceleration is found from a speed-time graph. [1]
The gradient of a speed-time graph gives the acceleration.

(b) Describe how the acceleration of the stone charges between point A and point D. [3]
From A to C, the acceleration of the stone decreases, as the gradient of the graph is decreasing.
From C to D, the acceleration of the stone is constant and negative (speed is decreasing).

(c) Explain, in terms of the forces acting, why the acceleration changes between point A and point B. [3]
When the stone is falling, there are two forces acting on it – its weight (downwards) and air resistance (upwards). As the stone's speed increases from A to B, its speed increases, thus increasing the upward air resistance. Since the weight of the stone stays constant, the resultant force downwards is reduced, thus resulting in a decreasing acceleration.

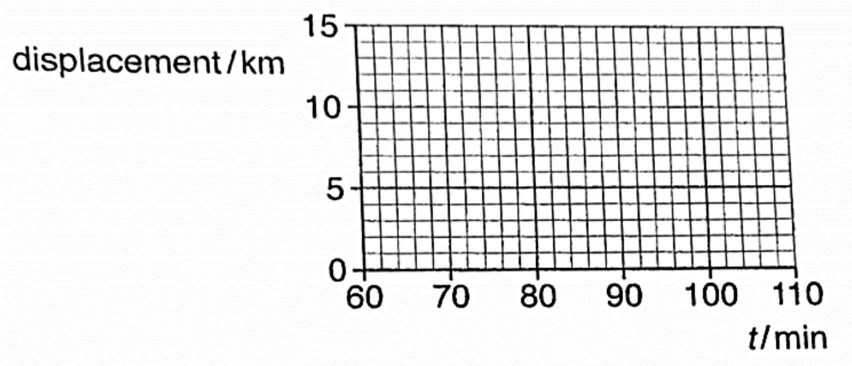
(d) Explain how the right figure above shows that the stone does not reach terminal velocity in the air. [1]
The stone hits the water at C. From the graph, the stone does not reach a constant velocity between A and C, thus showing that the stone does not reach terminal velocity in the air.

6. The figure below shows a tanker, which starts its motion from rest and travels in a straight line.



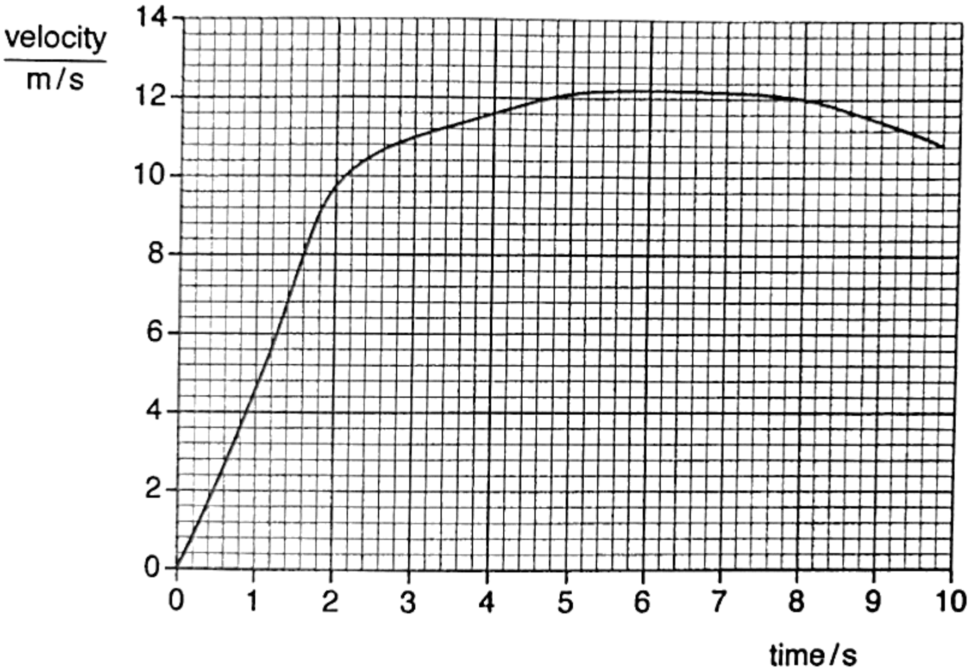
The tanker has a mass of 1.5 × 106 kg and the force acting forwards on the tanker due to the propellers has a constant value of 2.5 × 105 N. The acceleration of the tanker is not constant.
The engines stop at time t = 60 minutes. The velocity of the tanker then decreases. In the next 40 minutes, the tanker travels a distance of 12 km. At the end of this 40 minutes, the velocity of the tanker is zero.
(2018 P2A Q2c)

(a) On the figure below, sketch a graph to show the displacement of the tanker with time from t = 60 min to t = 110 min. Displacement is measured from the place where the engines stop. [2]



(b) State how the graph shows that the velocity of the tanker decreases. [1]
The gradient of the graph is decreasing.

7. The figure below shows the velocity-time graph of an athlete for a 100 m race. (2019 P2A Q1)



(a) State a time during the race when the acceleration of the athlete is largest. [1]
Any time from 1.2 to 1.8 seconds.

(b) The velocity of the athlete is always positive, but his acceleration is sometimes negative. Explain this statement. [2]

Positive velocity means that the athlete is still moving in the forward direction.
Negative acceleration in this case means that the athlete's velocity is decreasing.

(c) State how a student can use the figure above to show that the race is 100 m long. [1]

The area under the graph would give 100 m.

(d) Determine the average speed of the athlete during the race. [2]

The athlete took 9.8 s to run 100 m.

$$\text{Average velocity} = \frac{\text{total displacement}}{\text{total time taken}}$$

$$= \frac{100}{9.8} = 10.2 \text{ m/s}$$

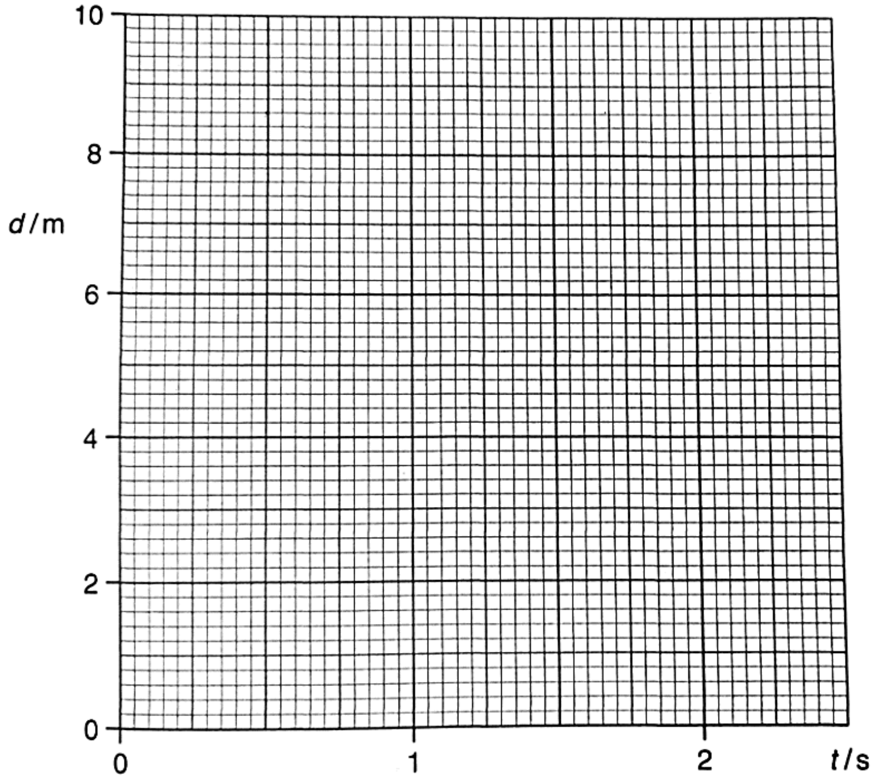
8. A badminton shuttlecock is a light, cone-shaped object with a large surface area.
In an experiment using electronic apparatus, a shuttlecock is released from rest and its distance d fallen is measured at different times t.
A mass is added inside the cone of the shuttlecock. The experiment is then repeated. The table below shows the results obtained with the light shuttlecock and with the heavy shuttlecock.

t / s	light shuttlecock d / m	heavy shuttlecock d / m
0	0	0
0.20	0.19	0.19
0.40	0.74	0.74
0.60	1.56	1.61
0.80	2.56	2.70
1.00	3.68	3.93
1.20	4.86	5.27
1.40	6.07	6.68
1.60	7.31	8.12
1.80	8.56	9.59
2.00	9.81	11.07

(a) Explain how the data in the table above for the light shuttlecock suggests that the speed is increasing at t = 0.40 s. [1]

The distance fallen from 0.20 s to 0.40 s is 0.74 – 0.19 = 0.55 m.
The distance fallen from 0.40 s to 0.60 s is 1.56 – 0.74 = 0.82 m.
Since the distance fallen increases for the same period of time, the speed of the light shuttlecock is increasing at t = 0.40 s.

(b) (i) On the figure below, draw a graph of *d* against *t* for the light shuttlecock. [2]



(ii) Explain how the graph shows that this shuttlecock reaches terminal velocity. [1]

The gradient of the graph becomes constant after 0.8 s.

(iii) Using the data in the table above, determine the terminal velocity of the light shuttlecock. [2]

Terminal velocity = Gradient of graph after 0.8 s

$$= \frac{9.81 - 3.68}{2.00 - 1.00}$$

= 6.13 m/s

(iv) Explain, in terms of the forces acting on the shuttlecock, why it reaches terminal velocity. [2]

Initially, the shuttlecock falls due to its own weight.

As its speed increases, the air resistance acting upwards increases until it is equal to the weight (downwards) of the shuttlecock. When this happens, there is no resultant force acting on the shuttlecock, and thus it reached terminal velocity (zero acceleration).

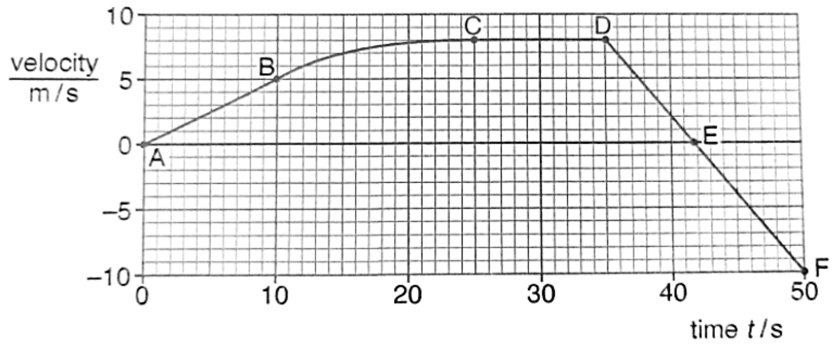
(c) (i) Compare the motion of the light shuttlecock with that of the heavy shuttlecock. [2]

Both shuttlecocks have the same initial acceleration, as indicated by the same distances fallen from 0 to 0.40 s. However, the heavy shuttlecock reaches terminal velocity at a later time (around 1.20 s). The terminal velocity of the heavy shuttlecock is also greater than the light shuttlecock.

(ii) Explain, in terms of the forces involved, the difference between the two sets of results. [1]

The heavy shuttlecock has more weight, which means it requires more air resistance acting against it in order for it to reach terminal velocity.

9. The figure below shows the velocity-time graph for a car initially travelling forward in a horizontal straight line. (2017 P2B Q10a)



Describe the motion of the car

(a) between A and B [1]

Constant acceleration. Speed increases uniformly from 0 to 5 m/s in 10 seconds.

(b) between B and C [1]

Decreasing acceleration. Speed increases non-uniformly from 5 to 8 m/s in 15 seconds.

(c) between E and F [1]

Negative constant acceleration. Speed increases in the opposite direction (increasing negative velocity) from 0 to -10 m/s in 8.5 seconds.