Chapter 5 – Turning Effects of Forces

Subject content

Content
- Moments
- Centre of gravity
- Stability

Learning outcomes
(a) describe the moment of a force in terms of its turning effect and relate this to everyday examples
(b) recall and apply the relationship moment of a force (or torque) = force × perpendicular distance from the pivot to new situations or to solve related problems
(c) state the principle of moments for a body in equilibrium
(d) apply the principle of moments to new situations or to solve related problems
(e) show understanding that the weight of a body may be taken as acting at a single point known as its centre of gravity
(f) describe qualitatively the effect of the position of the centre of gravity on the stability of objects

Definitions

Term	Definition	SI unit
Moment of force	Product of the force F and the perpendicular distance d from the pivot to the line of action of the force	Nm
Principle of Moments	When a body is in equilibrium, the sum of clockwise moments about a pivot is equal to the sum of anticlockwise moments about the same pivot	
Centre of gravity	Point through which whole weight of object appears to act	
Stability	Measure of ability of object to return to original position after it is slightly displaced	

Formulae

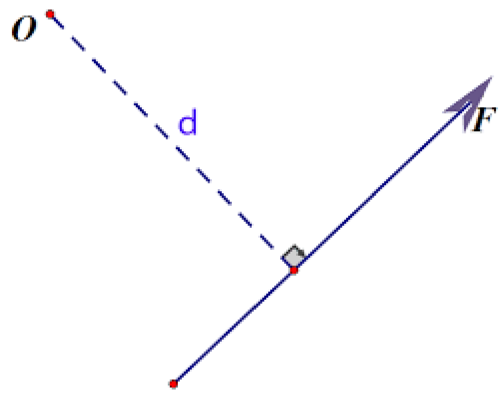
Moment of force	Equilibrium
$M = F \times d$	Σ clockwise moments = Σ anticlockwise moments Σ upward forces = Σ downward forces

5.1 Moments

Moment of force

$$\boxed{M = F \times d}$$

where M = moment of force (in Nm)
F = force (in N)
d = perpendicular distance from pivot (in m)



Vector quantity → need to state its
1. Magnitude (SI unit: Nm)
2. Direction (clockwise / anticlockwise)

5.2 Principle of Moments

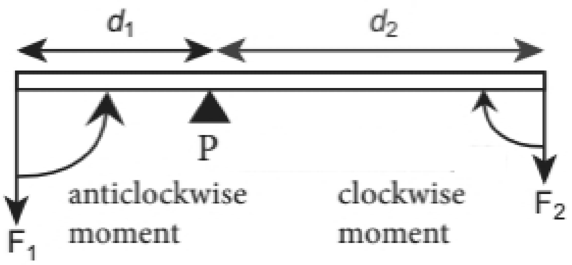
Principle of Moments

Total clockwise moment = Total anticlockwise moment

$$\boxed{F_1 d_1 = F_2 d_2}$$

Conditions for equilibrium

Object does not rotate
→ clockwise = anticlockwise moments (**balanced**)
1. resultant force on object = zero
2. resultant moment on object = zero



Calculation involving Principle of Moments

Steps:
1. Identify all **forces**, indicate magnitude + direction
2. Identify the **pivot**
3. Indicate **perpendicular distance** of line of action of force from pivot
4. Calculate **moments** (clockwise / anticlockwise) due to each force acting about pivot
5. Apply Principles of Moment: **Σ clockwise moments = Σ anticlockwise moments**
6. Apply **Σ upward forces = Σ downward forces** (where applicable)

Types of questions:

Figure	Pivot	Explanation

	middle of uniform metre rule (CG in middle)	• **Σ clockwise moments = Σ anticlockwise moments**
		• Need not account for moment created by weight of uniform metre rule (does not create moment)
		   o acting on pivot
		   o ⊥d from line of action of weight to pivot = 0

	not in middle of uniform metre rule	• **Σ clockwise moments = Σ anticlockwise moments** • Need to account for **moment created by weight of uniform metre rule** (not at pivot position)
	double pivots	• Take **moments about the other pivot** (so that force exerted at that pivot will not create a moment – no need to include in calculation) → find **vertical force acting upward on a pivot** • **Σ upward forces = Σ downward forces** for object in equilibrium
	anywhere along non-uniform wooden plank (CG not in middle)	• **Σ clockwise moments = Σ anticlockwise moments** • Need to account for **moment created by weight of plank**

5.3 Centre of Gravity

Centre of gravity

For object of regular shape and uniform density, centre of gravity → **geometrical centre**
(CG may lie outside of object)

Rectangle	Triangle	Circle
Cuboid	Sphere	Ring

Balance objects at CG
- ⊥**d** between pivot & line of action of W = 0
- **weight has no turning effect**

Find CG of irregularly shaped plane lamina
- Procedure
  1. make 3 small holes near edges
  2. suspend lamina freely from a pin
  3. hang plumb line from pin, in front of the lamina
  4. plumb line steady → trace the line on the lamina
- Precautions
  - holes must be small so that not too much weight of lamina is removed
  - laina free to swing about point of suspension
  - parallax error → view directly in front
- CG = **point of intersection** of three lines

5.4 Stability

<u>Types of equilibrium</u>

Types of equilibrium

1. **Stable equilibrium**
2. **Unstable equilibrium**
3. **Neutral equilibrium**

Stability	**Stable equilibrium**	**Unstable equilibrium**	**Neutral equilibrium**
Figure			
Centre of gravity when displaced	**Raised**	**Lowered**	**Same height**
Line of action of force	Falls **within base**	Falls **outside base**	Falls **within base**
Nature of moment about pivot (according to figure)	anticlockwise moment	clockwise moment	no moment
Motion of object	Return to original position	Continue to topple	Stay in any position to which has been displaced

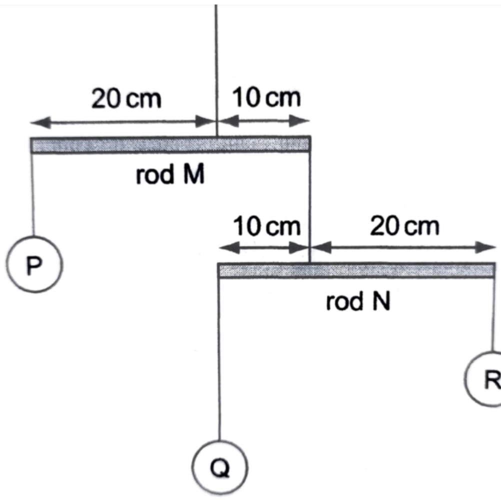
Increase stability → greater angle of tilt before line of action of weight falls out of base

1. **low centre of gravity**
2. **wide base area**

Typical questions

Multiple choice questions

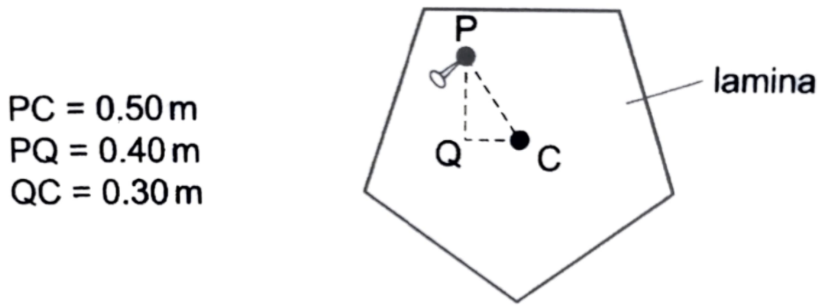
1 The diagram shows a decoration, which is made by suspending objects P, Q and R from light rods M and N. The masses of P, Q and R are such that the rods are horizontal.



Which row gives a possible combination of the masses of P, Q and R? (2011 P1 Q8)

	mass of P / g	mass of Q / g	mass of R / g
A	10	10	10
B	15	10	10
C	15	20	10
D	20	40	20

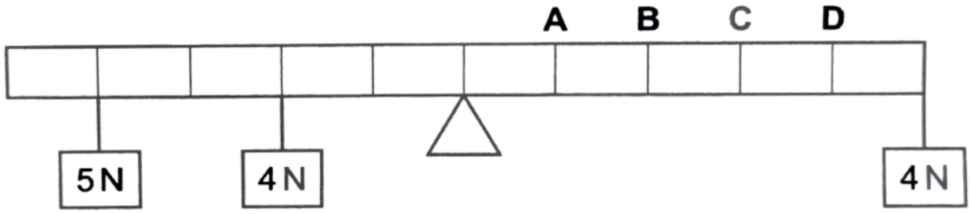
2 A lamina is freely suspended from point P.
The weight of the lamina is 2.0 N and the centre of gravity is at C. (2012 P1 Q8)



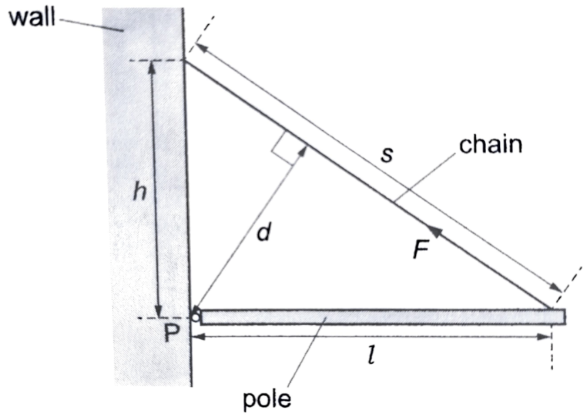
The lamina is displaced to the position shown.
What is the moment about P due to the weight of the lamina?

A 0.60 Nm clockwise
B 0.80 Nm anticlockwise
C Nm clockwise
D Nm anticlockwise

3 The diagram shows a beam. Without any weight, the beam balances at its centre.
Three weights are hung as shown.
At which point does a load of 2 N balance the beam? (2013 P1 Q11)



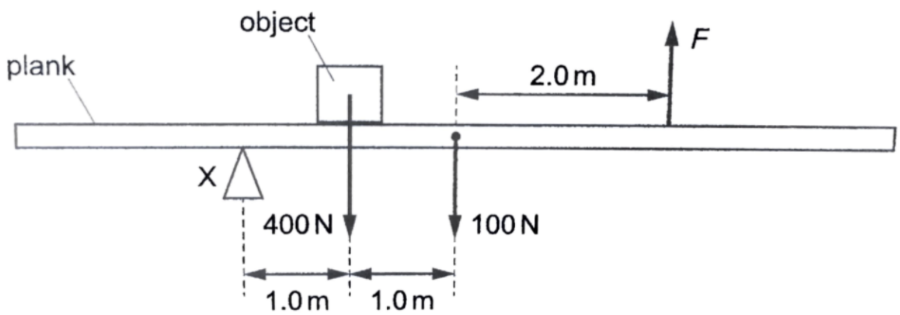
4 A horizontal pole is attached to the wall of a building. There is a pivot P at the wall and a chain is connected from the end of the pole to a point higher up the wall.



There is a tension force F in the chain.
What is the moment of the force F about the pivot P? (2018 P1 Q10)

A $F \times d$
B $F \times h$
C $F \times l$
D $F \times s$

5 An object is placed on a uniform plank with support at X.

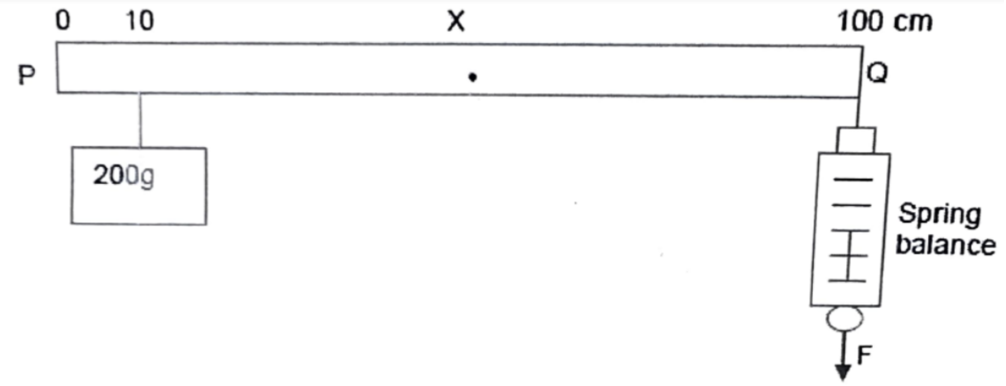


The object weighs 400 N and the plank weighs 100 N.
An upwards force F keeps the plank level. What is F? (2018 P1 Q11)

A 150 N
B 200 N
C 250 N
D 500 N

Structured questions

1 The figure below shows a uniform metre rule PQ that is pivoted at its centre X by a girl. A 200 g mass is placed at the 10 cm mark of rule. The rule is kept horizontal by a spring balance that is calibrated in Newton (N) and fastened at the other end Q of the rule.
(take g = 10 N/kg)



(a) Calculate the moment of force due to the mass hung. [2]

Moment due to 200 g mass = 2 N × 0.40 m
= 0.80 Nm

(b) Calculate the force the spring balance registers. [2]

F × 0.50 m = 0.80 Nm
F = 1.6 N

(c) State and explain how the value of the force registered by spring balance changes if the girl pushes up gently with her finger at the 40 cm mark, whereas the metre rule is balanced. [1]

The finger produces a clockwise moment, thus the clockwise moment required by the spring balance to balance the metre rule will decrease. Force will decrease.

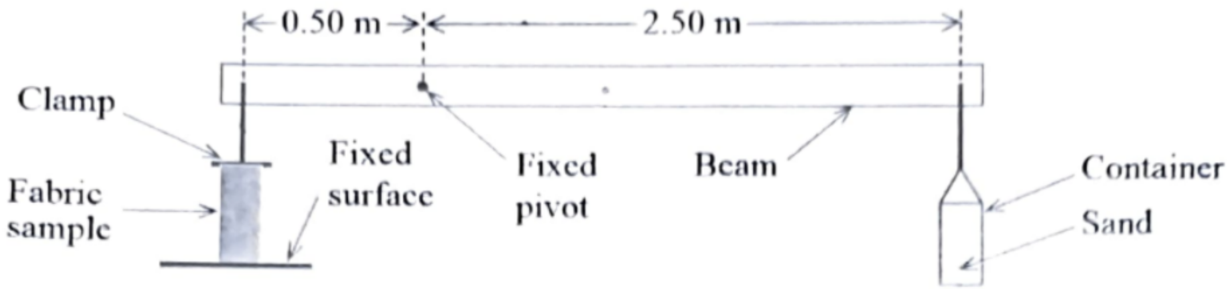
(d) Why was it necessary to know that the rule is pivoted at the centre? [1]

Moment due to weight of the metre rule will be zero as the perpendicular distance of weight to pivot is zero, thus we do not need to consider the weight of the metre rule.

(e) How would the reading on the spring balance change if the rule was pivoted not at its centre but at the 60 cm mark and kept balanced? Explain briefly. [2]

- Perpendicular distance of 2 N force to pivot will increase so anticlockwise moment due to 2 N increases.
- Anticlockwise moment due to the weight will also no longer be zero as its perpendicular distance is not zero.
- The perpendicular distance of *F* from pivot will also decrease.
- Thus, the reading of force at spring balance would increase considerably due to the Principle of Moments.

2 The diagram shows an arrangement which can be used to test the strength of a fabric. A sample of the fabric is placed between a fixed surface and a clamp. The clamp is attached to a uniform beam of weight 20 N. By adding sand to the container, a stretching force is applied by the clamp to the fabric. The force applied by the sand and container is 90 N.



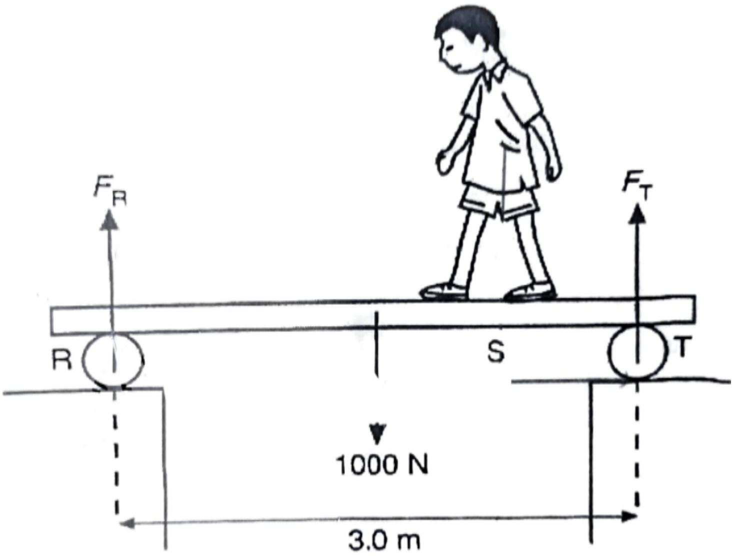
(a) Calculate the stretching force applied by the clamp to the fabric. [2]

Since beam is in equilibrium,
Σ anticlockwise moments = Σ clockwise moments
S × 0.50 m = 20 N × 1.0 m + 60 N × 2.50 m
S = 340 N

(b) State two ways in which the stretching force applied to the fabric could be increased using the same amount of sand. [2]

- Use a beam with a larger weight
- Attach the fabric closer to the pivot

3 The figure below shows a uniform beam used as a bridge and supported symmetrically by two rollers. A student of mass 50 kg stands at a distance of 1.0 m from T. (Take g to be 10 N/kg)



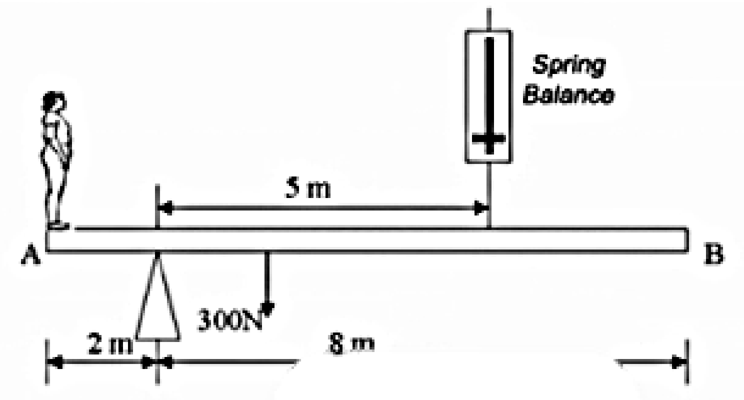
(a) Calculate the reaction F_T, by taking moments about R. [2]

Taking moments about R,
$F_T \times 3.0$ m = 1000 N × 1.5 m + 500 N × 2.0 m
F_T = 833 N (3 s.f.)

(b) Hence or otherwise, calculate the reaction F_R. [2]

Total upward forces = Total downward forces
$F_R + F_T$ = 1000 N + 500 N
F_R = 667 N

4 A non-uniform plank weighing 300 N is set up as shown. The spring balance reads 160 N when the woman stands at A, and 760 N when the woman stands at B.

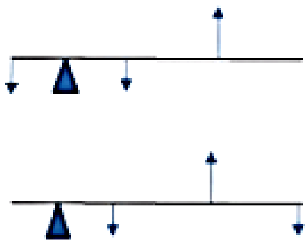


Calculate the weight of the woman. [3]

Answering technique

Step 1: Let weight be W and distance from 300 N to pivot be x – 2 unknowns

Step 2: Draw 2 free-body diagrams for boy at A and boy at B



Step 3: Form 2 simultaneous equations to solve for W and x.

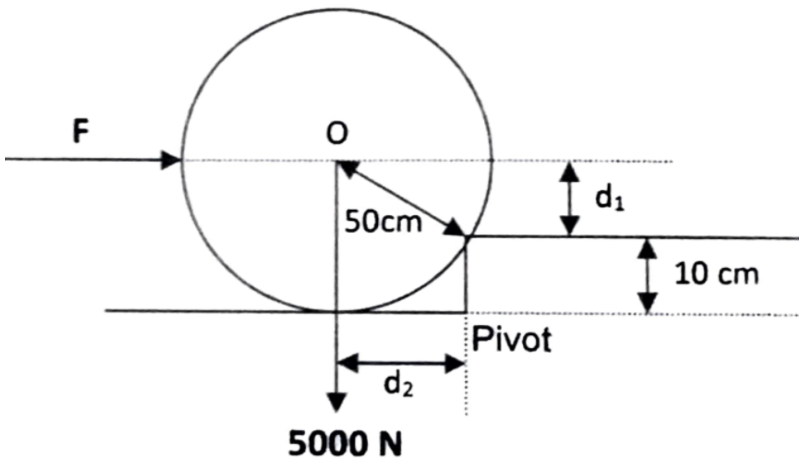
Let the weight of the woman be *W*
Let the distance from the pivot to centre of gravity of plank be *x*

When the woman is at A, taking pivot about the pivot point (so that R can be ignored):
Clockwise moments = $300x$
Anticlockwise moments = $160 \times 5 + 2 \times W$
At equilibrium, clockwise moments = anticlockwise moments
$300x = 800 + 2W$ ———— (1)
(Note: R is ignored because the perpendicular distance between R and the pivot is 0, i.e. moment of R about the pivot is 0. We eliminate unknown forces to reduce no. of unknowns.)

When the woman is at B, taking pivot about the same pivot point (so that R can be ignored):
Clockwise moments = $300x + 8W$
Anticlockwise moments = $760 \times 5 = 3800$
At equilibrium, clockwise moments = anticlockwise moments
$300x + 8W = 3800$ ———— (2)

Solving the two simultaneous equations gives us
W = 300 N

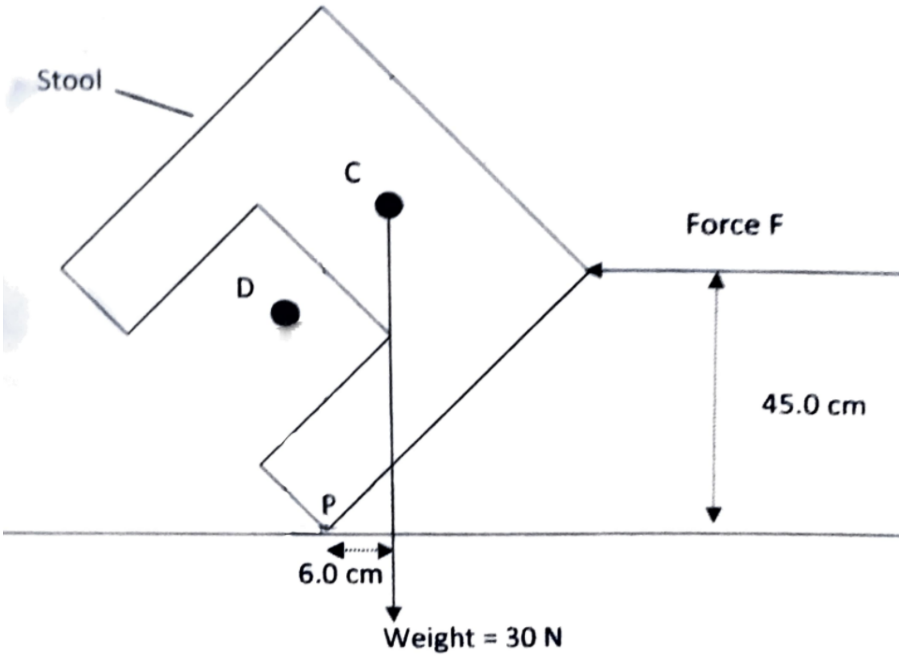
5 The diagram shows the cross-sectional view of a log being pushed up a step. The log has a diameter of 1.0 m and a weight of 5000 N. What is the minimum value of the force *F* which will just lift the log? [2]



$d_1 = 50 - 10 = 40$ cm
$d_2 = \sqrt{50^2 - 40^2} = 30$ cm

When the log is just lifted up, it pivots about point P.
Taking moments about P,
Σ clockwise moments = Σ anticlockwise moments
$F \times d_1 = W \times d_2$
$F \times 0.40$ m $= 5000$ N $\times 0.30$ m
$F = 3750$ N

6 The figure shows a stool. A horizontal force F keeps the stool balanced. C is the centre of gravity of the stool and the weight of the stool is 30 N.



(a) On the figure above, draw an arrow to show the vertical force that acts upwards on the stool and indicate its magnitude. [2]

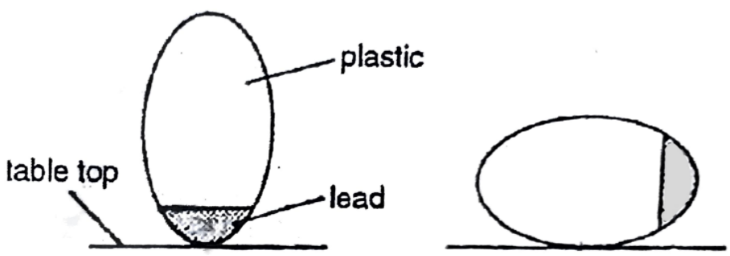
(b) Calculate, using the distances marked on the figure above, the value of the force F. [2]

30 N × 0.06 m = F × 0.45 m
F= 4.0 N

(c) Explain why a stool is more stable with a centre of gravity at D rather than at C. [2]

- <u>With a lower centre of gravity, the line of action of the weight will stay within the base area of the stool for a larger angle of tilt, hence it is more stable.</u>
- <u>The weight will create an anticlockwise moment about the pivot, which will restore the stool back to the upright position.</u>

7 The left figure below shows a cross-section through a child's toy called a Tumbling Kelly. The toy consists of a thin plastic egg-shaped shell with a layer of lead fixed in the bottom. When the toy is knocked over as shown in the right figure, it always rolls back to the upright position.



(a) On the left figure, use small crosses to mark the positions of the centre of mass of

(i) the plastic shell; label the cross P, [1]

(ii) the lead; label the cross L. [1]

(b) On the right figure, mark and label the external forces acting on the toy. [1]

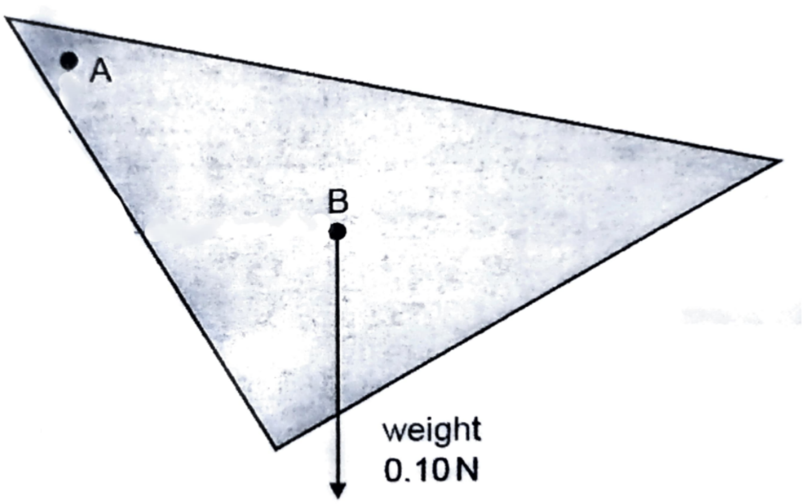
(c) Explain why the toy returns to the upright position. [2]

- When the Tumbling Kelly is knocked over, the centre of gravity of the toy is raised.
- The weight of the toy acting through its centre of gravity creates a clockwise moment about the pivot.
- Hence, the toy rotates clockwise due to its moment and returns to the upright position.

(d) What are the properties of plastic and lead which makes them suitable materials for use in a Tumbling Kelly? [2]

In terms of mass, plastic is lighter than lead. This results in the overall centre of gravity of Tumbling Kelly being lowered. A lower centre of gravity increases the stability of the toy.

8 The figure below shows a sheet of metal suspended from a hole in one corner at A. The weight of the metal is 0.10 N and the centre of gravity is at B. The diagram is drawn in full scale.



(a) The sheet turns because of the moment of the weight about point A. Using a measurement taken from the figure above, calculate the moment of the weight about point A. On the figure above, indicate the measurement that you made. [3]

M = 0.10 N × 3.6 cm
= 0.36 Ncm

= 0.0036 Nm

(b) The sheet in the figure above swings freely and comes to rest.

(i) Sketch a diagram of the sheet in its final rest position. [1]

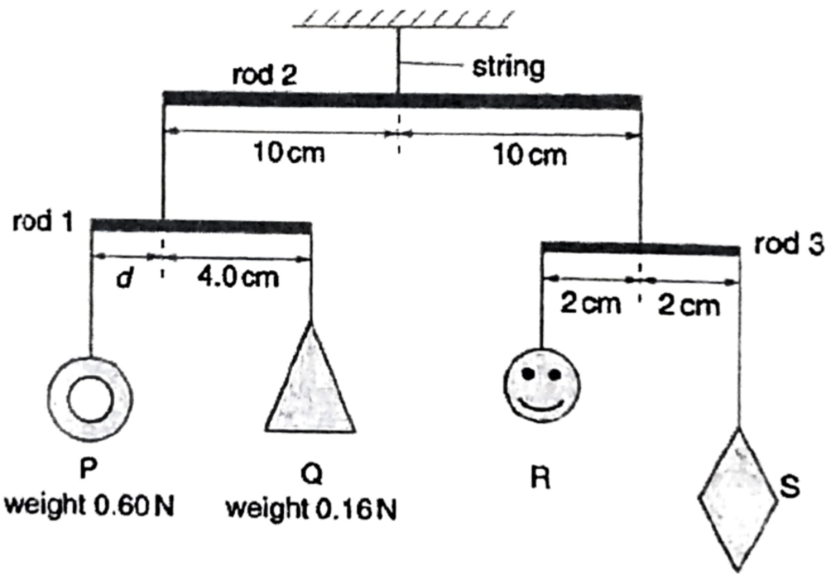
(ii) Explain why the sheet remains at rest in this position. [2]

- The centre of gravity lies vertically below pivot point A.
- The moment of weight about A is zero.

(c) The sheet is placed on a table. State 2 reasons why it is more stable when it is placed flat on the table than when it is placed on the table in a vertical position. [2]

The centre of gravity is lowered and has a larger base area.

9 The figure below shows a decoration that hangs by strings from the ceiling in a toyshop. Four objects P, Q, R and S hang by strings from rods 1 and 3. These two rods hang by strings from rod 2. The decoration is completely stationary and the rods are horizontal. The weights of the rods and the strings may be neglected.



Object P weighs 0.60 N and object Q weighs 0.16 N.

(a) Calculate the value of the distance *d* on the figure above. [1]

0.60 N × *d* = 0.16 N × 4.0 cm
d = 1.07 cm (3 s.f.)

(b) Determine the weight of object R and object S. [2]

$(W_P + W_Q) \times 10$ cm = $(W_R + W_S) \times 10$ cm
$W_R + W_S = 0.76$ N

$W_R \times 2$ cm = $W_S \times 2$ cm
$W_R = W_S$

Weight of object R = 0.38 N
Weight of object S = 0.38 N.

(c) Object Q falls off. Using moments, state and explain what happens to rods 1 and 2. [2]

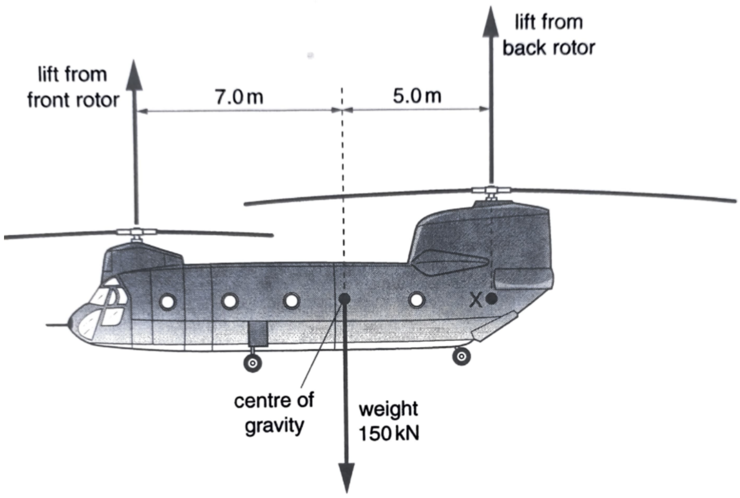
(i) rod 1:

<u>There is no force acting on the right of the pivot, so clockwise moment = 0.</u>
<u>Only an anticlockwise moment caused by P exists, so rod 1 rotates anticlockwise about the pivot.</u>

(ii) rod 2:

<u>The weight on the left of the pivot is smaller than the right.</u>
<u>The total clockwise moment is larger than the total anticlockwise moment, so rod 2 rotates clockwise about the pivot.</u>

10 The figure below shows a helicopter stationary in the air.
Vertical lift forces are produced by the front rotor and by the back rotor.



The weight of the helicopter is 150 kN. Horizontal distances are marked on the figure above.
(2014 P2A Q3)

(a) Describe the difference between mass and weight. [1]

Mass refers to the amount of substance in an object

Weight refers to the gravitational force acting on the object.

(b) Determine the mass of the helicopter. The gravitational field strength g is 10 N/kg. [1]

$W = mg$

$m = \frac{W}{g} = \frac{150 \times 10^3}{10} = 1.5 \times 10^4$ kg

(c) By taking moments about point X, calculate the lift force from the front rotor. [2]

Taking moments about X,
clockwise moments = anticlockwise moments
$\text{Lift}_{\text{front}} \times (7.0 + 5.0) = (150 \times 10^3) \times 5.0$
$\text{Lift}_{\text{front}} \times 12 = 75 \times 10^4$
$\text{Lift}_{\text{front}} = 6.25 \times 10^4$ N

(d) Calculate the lift force from the back rotor. [1]

As the helicopter is stationary, the forces are balanced:

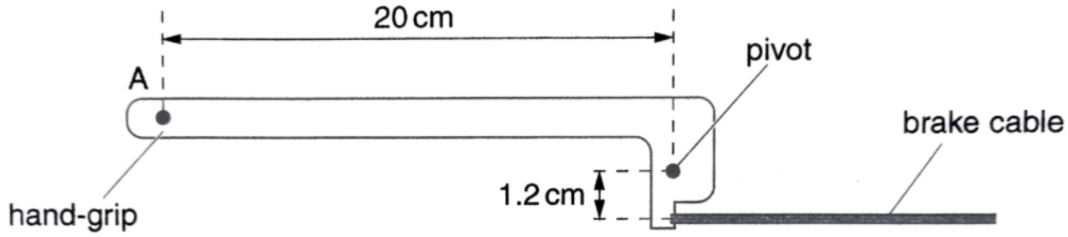
total downward force = total upward force

150 kN = 6.25 kN + $\text{Lift}_{\text{back}}$
$\text{Lift}_{\text{back}}$ = 150 – 62.5 = 87.5 kN

(e) The helicopter pilot adjusts the lift forces at the front and back of the helicopter. The front of the helicopter tilts down, whilst the centre of gravity of the helicopter stays at the same height. State how the lift forces from the rotors are adjusted to achieve this effect. [1]

For the front of the helicopter to tilt downwards, the lift from the front rotor must be reduced, while the lift from the back rotor is increased, in order to keep the centre of gravity at the same height. (Note: As a result of this change, the helicopter will no longer be stationary, and will move forward.)

11 The figure below shows a simplified diagram of the handbrake lever of a car. Distances are marked on the figure.



A force of 30 N is applied by the car driver at point A. (2015 P2A Q2)

(a) Describe how, with a 30 N force at A, the driver can produce the largest moment about the pivot. [1]

With a 30 N force at A, the perpendicular distance of the line of action of the force from the pivot is the greatest. Thus, using the formula Moment = $F \times d$, this would generate the largest moment about the pivot.

(b) Calculate the largest moment of this force about the pivot. [2]

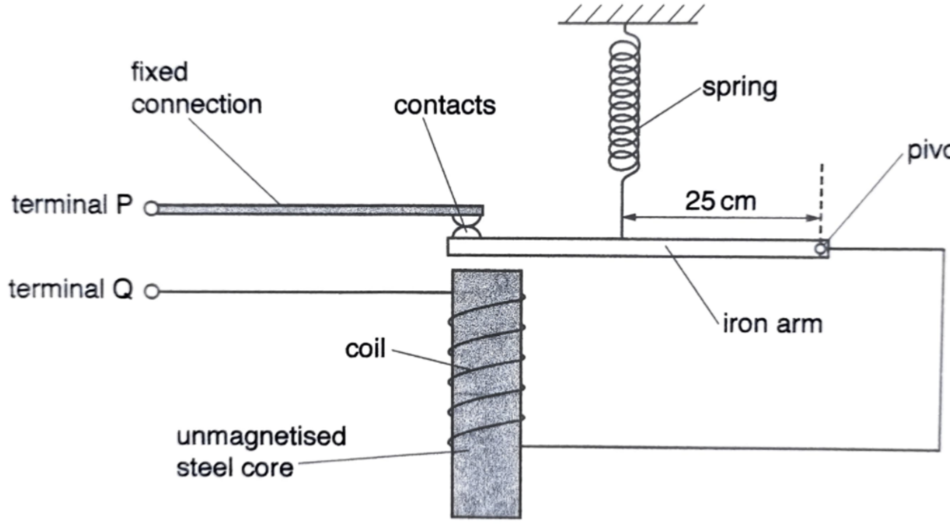
Moment = $F \times d$
= 30 N × 0.20 m
= 6 Nm

(c) Calculate the largest force in the brake cable. [1]

Moment at the brake cable = 6 Nm
Largest force in the brake cable:

$F = \frac{M}{d} = \frac{6}{0.012} = 500$ N

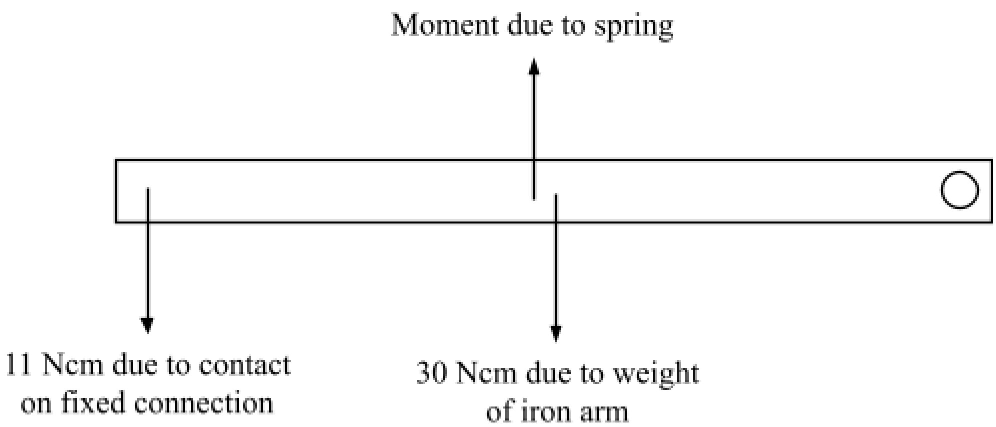
12 A student constructs a model of a circuit breaker using an unmagnified steel core, as shown in the figure below. The distance from the spring to the pivot is 25 cm.



The iron arm can move freely about the pivot. The weight of the iron arm produces a moment of 30 Ncm about the pivot. The contact on the fixed connection pushes downwards on the arm and produces a moment of 11 Ncm about the pivot.

Calculate the force exerted by the spring on the arm. [2]

(2016 P 2A Q8a)



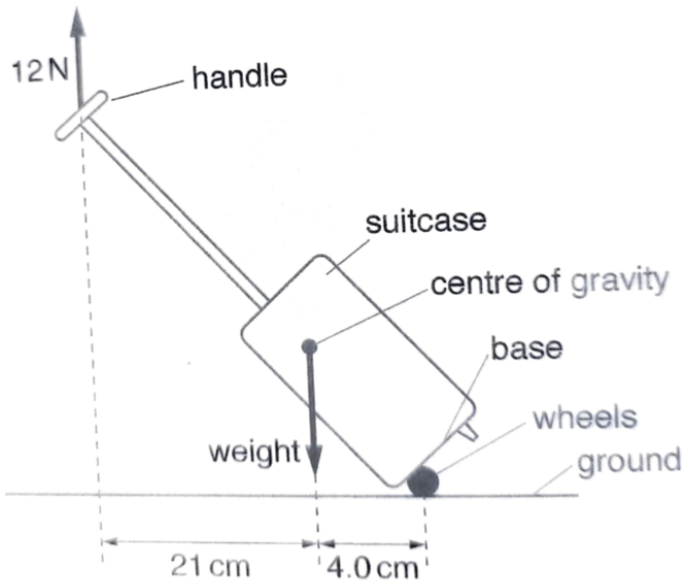
Considering the Principle of Moments:

Σ clockwise moment = Σ anticlockwise moment

$F_{spring} \times 25 = 11 + 30$

$F_{spring} = 1.64$ N

13 The figure below shows two forces acting on a stationary suitcase. The 12 N upward force on the handle keeps the suitcase balanced. The wheels act as the pivot.



(a) By taking moments, determine the weight of the suitcase. [2]

Weight of object:
W = mg = 0.25 × 10 = 2.5 N
(Note: Mass needs to be in kg.)

Moment = F × ⊥d = 2.5 × 0.30 = 0.75 Nm
(Note: We use distance AX, as the weight is vertical, hence the perpendicular distance from the pivot would be the horizontal distance from the pivot A.)

(b) The suitcase is repacked. All the heavier contents are placed near the base. Explain why the suitcase is now more stable. [2]

<u>The centre of gravity is lowered.</u>

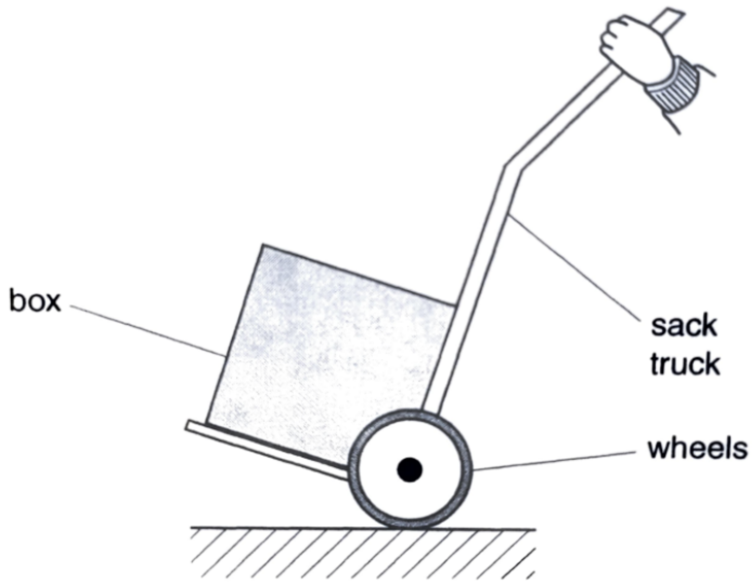
<u>This means that the line of action through its weight is more likely to be within the base of the suitcase, making it more stable.</u>

(c) Describe one force acting on the suitcase that is not shown on the figure above. [2]

<u>Normal reaction force acting by the ground on the suitcase.</u>

14 (2011 P2B Q11 OR)

(a) The figure below shows a sack truck supporting a box filled with sand.



Three of the forces acting on the truck are
- the weight W of the box
- the effort force E provided by the hands
- the force F between the ground and the wheels

(i) On the figure above, mark and label these three forces. Show clearly where each force acts and the direction of each force. [3]

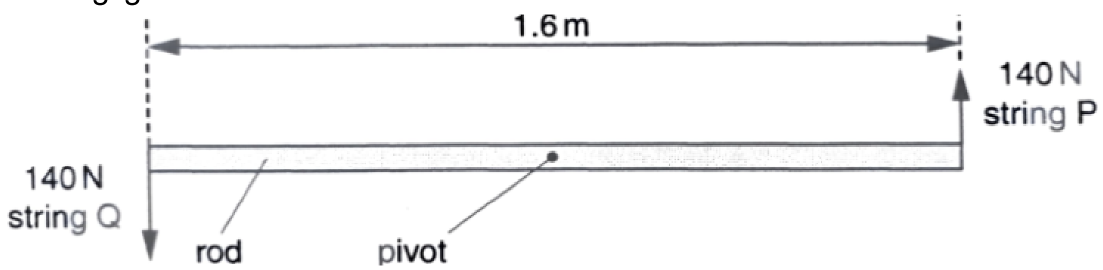
(ii) State the principle of moments. [1]

The Principle of Moments states that for an object in equilibrium, the sum of the clockwise moments about a pivot is equal to the sum of the anti-clockwise moments about the same pivot.

(iii) Explain how the design of the truck makes it easier to lift the load. [2]

The perpendicular distance of the effort applied from the pivot is further away, compared to the perpendicular distance of the weight / load from the pivot. As such, a smaller effort is required to produce a clockwise moment in order to balance / lift an object of larger weight.

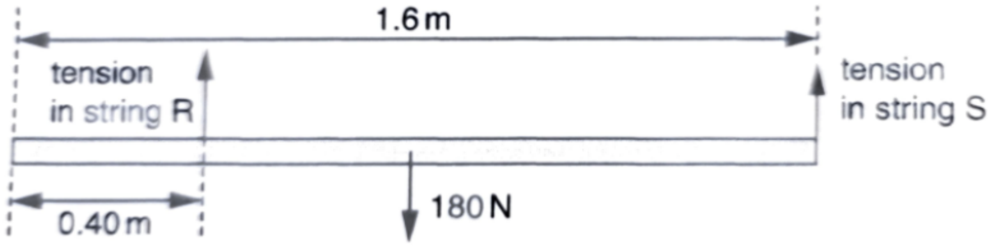
(b) The figure below shows two strings P and Q that exert vertical forces of 140 N at each end of a horizontal rod of length 1.6 m. The rod is pivoted at its centre and the weight of the rod is negligible.



Calculate the total moment acting on the rod about the pivot. [1]

Total moment = (140 × 0.8) × 2
= 224 Nm (anti-clockwise)

(c) Another uniform rod of length 1.6 m is supported by two strings R and S. This rod has a weight of 180 N which acts at the centre of the rod. String S is at one end of the rod and string R is 0.40 m from the other end, as shown in the figure below.



Calculate the tension in string R. [3]

Taking S as the pivot,
clockwise moments = anticlockwise moments
R × (1.6 - 0.40) = 180 × (1.6 ÷ 2)
R = 120 N
∴ The tension in R is 120 N.